Data-Driven Auditing of Business and Self-Employment Earnings

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Abstract

There is increasing reliance on data-driven auditing of businesses and proprietorship. However, the tax returns have garbled signals that are further confounded due to underreporting. Entrepreneurs' profits depend on their individual types and a common market shock. A high ability, high profit earner underreports only when he observes his neighbor to have earned low profits: neighborhood information about the performance of other entrepreneurs in the same business prompts such strategic reporting, making the volume statistic of 'high submissions' a meaningful indicator of market shock. In response auditors scrutinize all low profit returns only if the proportion of high submissions exceeds a threshold cutoff. Because this cutoff is endogenous and depends on the stochastic types and market shock, tax returns cannot systematically avoid audit scrutiny as in exogenous cutoff tax returns models. Auditing is enriched to combat the high 'tax gap,' a well-known problem in tax enforcement.

JEL Classification Numbers: H24; H26; D8; K4

Key Words: Tax evasion, market shock, neighborhood information, volume statistic, bounded rational learning, auditing without commitment, cutoff strategy, gross tax gap.

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1 Introduction

A document prepared by the IRS (2019) titled, "Tax Gap Estimates for Tax Years 2011-2013," reports a *gross tax gap* of \$441 billion with the estimated total true tax \$2,683 billion.[1] After enforcement by audits and other late payments, the *net tax gap* amounts to \$381 billion, so the net compliance rate is 85.8%. Clearly, enforcement, for lack of enough audit budget and imperfect audits, still leaves out 14.2% of total legal tax uncollected. Of the gross tax gap, individual income tax make up \$314 billion which is roughly 71%. Further, underreporting of 'business income' contribute 25% of the gross tax gap; see Table 2 of the IRS report.

For Canada, Schuetze (2012) reports that

> "*Between 1981 and 1998 the percentage share of self-employment in total employment increased from 12.8 percent to 17.1 percent. Nearly 2.5 million Canadians reported working for themselves in 1998 compared to just 1.4 million in 1981 - a 68 percent increase. Over the same period the number of wage employees in Canada grew by only 19 percent. While still significant, the contributions of self-employment to overall declared employment incomes in Canada did not match the contributions entrepreneurs made to total employment. In 1998, the most recent year for which there is data, 7.5 percent of total declared employment income or 33 billion dollars in income was derived from self-employment.*"

The above paragraphs highlight the extent of tax noncompliance by businesses and the self-employed in the USA and Canada. The purpose of this study is to enrich our understanding of this noncompliance, and suggest what the authorities can do about it.

We propose a hypothesis that the IRS, and more generally the tax authorities in many countries including developing economies, lack enough information about specific business conditions, or what we term as sector-specific 'common shocks'. This information is diffused and can be gleaned from submitted annual tax returns. If the tax authorities learn from the tax data and accordingly tailor auditing strategy, enforcement becomes much richer. The basic message of our analysis is a simple one: so long as the proportion of high tax returns exceeds a threshold, the tax authorities should consider the underlying business conditions to

[1] The gross tax gap has three components: nonfiling, underreporting, and underpayment.

be favorable and so will wisely spend their enforcement dollars to audit those submitting low returns. For our prescribed strategy to work, however, the tax department or the relevant government authorities should be willing to raise sufficient enforcement dollars.[2] As such we will have nothing to add to the issue of enforcement budget.[3] Formulating purely as a cost–benefit exercise but without the restriction of an exogenous budget, our analysis should be viewed for its policy guidance on how best to audit the self-employment and business sectors.

The framework studied in this paper can be taken to many unregulated sectors, in developing countries in particular, where it is too costly for the tax authorities to keep track of the local conditions. For example, in agriculture it is not enough to know whether the crop season has been good or bad, one must also know the informal labor market conditions, the market for the final crop due to the influence of intermediaries in hoarding the supply of crops etc., to correctly estimate the general influence of common shock on the profits of wealthy farmers with large landholdings. What weather the fishermen encountered throughout the year in deep sea fishing or regular fishing trolls may not be known, first hand. Even in semi-formal services and trades in developed countries, e.g., construction, individual consulting and counselling, restaurants, events organizing, private transport, property management, etc., buyers and sellers can engage in undocumented transactions, so the tax authorities can be unsure of the general health of any specific business.

[2]In the case of the United States the chronic underfunding of the IRS since 2010 is a well-known problem. During the period 2010-2018 the enforcement budget dropped by 24%; see Figure 1 of the Center on Budget and Policy Priorities (2020) document. The net tax gap of \$381 billion discussed at the start is likely to be a combination of the IRS's inadequate enforcement budget and the undetectable component of incomes.

[3]So long as the net marginal return of relaxing the budget constraint by a dollar, measured by increased compliance plus the extra recovery from evaded tax and the penalties (net of the borrowed dollar), exceeds the market rate of interest, there is no reason why the government cannot borrow from the capital market the additional funds needed for enforcement. A recent document, "An Analysis of Certain Proposals in the President's 2022 Budget" (see Congressional Budget Office, 2021), estimates that the Administration's proposal to increase IRS funding for enforcement and related operations support by \$60 billion over the 2022-2031 period would increase revenues by approximately \$200 billion over those 10 years. To quote from the document, "CBO's estimate of revenues is based on the IRS's projected returns on investment (ROIs) for spending on new enforcement initiatives. The IRS estimates those ROIs by calculating the expected revenues that would be raised from taxes, interest, and penalties as a result of the new initiatives and dividing them by their additional cost. ... In recent years, peak ROIs have ranged from 5 to 9. That is, a \$1 increase in spending on the IRS's enforcement activities results in \$5 to \$9 of increased revenues."

In formally modelling taxpayers—tax authority strategic interactions, we consider an n number of entrepreneurs who draw their profits, high (1) or low (0), based on an intrinsic type, high (h) or low (ℓ), privately known to the entrepreneurs and a common market shock ϵ that is favorable ($\epsilon = 1$) or unfavorable ($\epsilon = 0$). Each entrepreneur observes only another entrepreneur's profit and each entrepreneur's profit can be observed by exactly one other entrepreneur but with no mutual information. That is, entrepreneurs are located in a circle with information flow in a counterclockwise direction as in Fig. 1. With this information structure, each taxpayer submits a profit report based on his own ability draw and own and neighbor's profit information.

We show one simple central result: if the tax authority receives m high-profit returns exceeding a threshold $\hat{m}$, it will audit all low returns, and otherwise it will audit no submission. Translated in the language of the recent tax-gap debate, the optimal audit ensures that the *estimated gross tax gap (net of underpayment) plus expected fines*,[4] call it **recoverable**, covers the cost of auditing $n - m$ low submissions for $m \geq \hat{m}$, where n is the number of entrepreneurs; for $m < \hat{m}$ audit costs will exceed the recoverable.[5] The number m helps the tax authority to estimate the likelihood of a favorable market shock using the taxpayers' submission strategies. The taxpayers will choose to truthfully submit high-profit reports only when their individual types are low and both their own and their neighbors' profits are high, and all else submit low-profit reports that can be truthful or nontruthful. The idea behind such tax return submission is that, a high-ability entrepreneur always attributes his high profit to his good skills and take a chance at evasion even if the neighbor's profit is high, whereas a low-ability entrepreneur views both his and neighbor's high profits indicative of a favorable market shock. These strategies lead the tax authority to believe, when m is

[4] We interpret underpayment to be declared by the taxpayer and thus known to the tax department, which will eventually be paid with interest. Put differently, we ignore underpayment due to genuine mistakes in taxpayer's understanding of the tax owed. See footnote 1.

[5] We will clarify later in the paper that our analysis assumes that an audit will detect underreporting with probability 1. That is, we abstract away from any undetectable component of self-employment incomes. The qualitative nature of our analysis should hold in the presence of an undetectable component. Where detection probability is less than 1, optimal audit should recommend audits of all low submissions so long as the expected tax recovery plus expected fines cover the overall cost of audits.

reasonably high, that the market shock is favorable with a high probability especially with all high reports coming from low-ability entrepreneurs only, triggering audits of low-profit submissions.

The cutoff strategy audits, and more generally a commitment to an audit rule, appeared in the works of various authors in the early income tax evasion literature (e.g., Reinganum and Wilde, 1985 and 1986; Cremer et al., 1990). Most of these works assumed the cutoff strategy to be exogenous that is often problematic due to the fact that those who are audited do not lie while those who are not audited just report the threshold income level. Ours is in terms of an aggregative index, the number of high reports ($\mathfrak{m}$), that comes out from the model endogenously as an equilibrium data. The potential evaders thus cannot predict exactly what $\mathfrak{m}$ will prevail and hence cannot fine-tune the binary evasion/non-evasion decision even though they can reason that the tax authority will use the cutoff $\hat{\mathfrak{m}}$. Viewed from this angle, our model gives rise to heterogeneity in tax evasion behavior: two entrepreneurs with the same high profits may choose different submissions, one truthful and another nontruthful. In turn, the message content of $\mathfrak{m}$ is garbled which is to be expected for any aggregate market data. Yet the result echoes with how tax authorities might behave in actual auditing: if market environment seems favorable it is more likely to audit low submissions than if market were unfavorable. The result is also derived in a very stripped-down model with no news reporting of the state of the economy. Clearly, any extraneous information on the economy via media reports will influence strategies in a predictable manner: vibes of a favorable economy will make potential tax evaders less prone to taking risks whereas negative vibes will incline them towards greater evasion, and in parallel the tax authority will be more likely to audit in the former case and less so in the latter case. We do not pursue this extension.

Fundamental to our equilibrium construction are two assumptions. First, unlike many models of tax enforcement we assume away enforcement budget. Acknowledging that limited budget can cripple auditing, our main point is simple – why not outsource auditing to an independent agency based on commissions when large 'tax gap' clearly shows money to be made by relaxing the budget?[6] Of course the agency must be subject to strict checks and balances given the confidential nature of information relating to one's business and the right to privacy of the subjects of audits. In a related work (Bag and Wang, 2021), a similar approach to auditing has been analyzed. The second assumption considers the tax authority to be 'boundedly rational'. That is, it estimates market shock principally based on the value of $\mathfrak{m}$ and not doing any further updating using an iterative Bayesian process for all those entrepreneurs submitting low returns. Bounded rationality is a well-accepted approach in economic theory pioneered principally by behavioral and experimental economists (see surveys by Crawford (2013), Rabin (2013) and even earlier by Conlisk (1996)). Conlisk notes: "A decision maker who finds optimization impossible or unduly costly may instead solve a simpler, approximate optimization problem" (Conlisk, 1996, page 676). Conlisk further writes, "Near rationality models suggest that the benefit of upgrading from bounded to unbounded rationality may be small. At the same time, computational complexity models suggest that the deliberation cost of upgrading may be sizable, even astronomical" (page 679). Also, see Young (2004): *Strategic learning and its limits*.

Finally, our work should be useful for its guidance on tax audit policy specifically for businesses and sole proprietorships. It is a reasonable guess that the tax authorities use sophisticated data-intensive algorithms that determine what will be audited.[7] In this regard, the use of any available information about the current state of particular businesses should be relevant. While there is no official disclosure on how tax audits are conducted in the USA or other countries, our web search yields useful evidence that lend support to our modelling, with the audit agencies relying on: (i) intelligence gathering and "information received from businesses and members of the public"; (ii) comparison of tax filings against

[6] This idea of 'making money' from auditing is consistent with the standard objective of maximizing tax revenue net of audit cost. In defense of tax revenue as an objective of the IRS Scotchmer (1998) writes (p. 229), "This is a reasonable assumption in this age of budget deficits, since an enforcement agency would be replaced if it left unexploited opportunities to enhance revenue." See also Cremer et al. (1990). Different from these authors is our relaxation of the assumption of an exogenous audit budget.

[7] As an example, see Battaglini et al. (2020) for tax audits of sole proprietorship in Italy; refer footnote 10 in the literature review of this Introduction.

industry based average taxes to determine suspect targets for audits; (iii) predictive analytics to determine whether client's financial data conform to what are known about companies in comparable circumstances, etc.[8] All of these strongly suggest more or less a common data-oriented learning/inference approach. In the academic research on accounting and auditing, the importance of the auditor's awareness of business environments and strategies is also well recognized (Power, 2007).[9] More generally, using market-level information to filter out common shocks and better assess individual behavior is well-known in the context of regulation (yardstick regulation) and compensation (relative performance evaluation); Shleifer (1985), Sobel (1999), Aggarwal and Samwick (1999), Celentani and Loveira (2006).

Related literature. The work connects with the extensive literature on general tax noncompliance and enforcement – Allingham and Sandmo (1972), Reinganum and Wilde (1985), Graetz et al. (1986), Border and Sobel (1987), Melumad and Mookherjee (1989), Cremer et al. (1990), Chander and Wilde (1998), to mention a few. More directly relevant are the works on tax evasion in hard-to-regulate self-employment activities (Pissarides and Weber, 1989; Ihriga and Moe, 2004; Blackburn et al., 2012; Dabla-Norris et al., 2008; Bigio and Zilberman, 2011; Bag and Wang, 2021).

[8] See https://www.revenue.ie/en/corporate/assist-us/reporting-shadow-economy-activity/what-revenue-is-doing.aspx. A (July 25, 2021) report in *The Economic Times* in India has the following: "Several companies, individuals get tax notices as data analytics uncovers gaps in filings" at https://economictimes.indiatimes.com/news/india/several-cos-individuals-get-tax-notices-as-data-analytics-uncovers-gaps-in-filings/articleshow/84738929.cms?from=mdr. *Australian Tax Office* states in their headline report titled, The cash and shadow economy, that "Benchmarks are one of the tools we use to help us detect businesses that may be avoiding their tax obligations, particularly cash transactions." (see https://www.ato.gov.au/general/gen/the-cash-and-shadow-economy/.) *KPMG/Forbes Insights* posted an article titled, Three Technologies That Will Change The Face Of Auditing, dated July 16, 2018, and makes the following observations: "Specifically, auditors can use client data – and combine it with industry or market data – to enable a deeper and more robust understanding of the state of the business and any risks."; "External auditors working with a client can use predictive analytics to assess whether the client's financial or other data conform to the expected norms for comparable historical data from both within the company as well as from companies in comparable circumstances." (https://www.forbes.com/sites/insights-kpmg/2018/07/16/three-technologies-that-will-change-the-face-of-auditing/?sh=48193f3a7544.)

[9] In his editorial Introductory review of *Business Risk Auditing* the author writes, "The proposed systems approach by KPMG...adopted and adapted elements of the COSO framework and positioned auditors as business risk assessors, building on and extending common understandings of auditors' need to know the business."

We propose a new method to analyzing tax auditing/enforcement. None of the previous works, including the ones mentioned above, address learning induced auditing.[10] While it is a novel application, that the market data can be utilized in the best interests of selected parties is an old idea. Two papers stand out for a general guidance to how we approach our problem: Blume et al. (1994), and Bala and Goyal (1998).

Blume et al. analyze a rational expectations competitive market for a risky asset to show how the volume of trade transmits valuable information about the asset fundamentals that are dispersed among the traders. The authors consider repeated trading of the asset by a finite number of traders. Even as the number of traders becomes very large, prices alone cannot convey the relevant information fully. A trader who watches both prices and volume generally does better than the one who ignores the volume statistic. While our tax auditing model is far from the rational expectations model of asset trading, the gleaning of information about the market shock (in the form of percentage of high-profit reports in overall submissions) through the taxpayers' observation of their neighbors' realized profits bears a resemblance to Blume-Easley-O'Hara's concept of learning from trade volumes.

Bala and Goyal study a rich setting of social learning by agents in a connected society through the experiences not only of their own but that of their neighbors. They analyze how neighborhood information structure facilitate or hinder social adoption of an optimal action when agents face similar type of decisions. The bounded rationality approach adopted by Bala and Goyal imposes a restriction on agents' belief formation similar to the one in the current paper: "...in updating her beliefs, an agent does not make inferences concerning the experience of unobserved agents (such as some of the neighbours of her neighbours), from the choice of actions of her neighbours" (page 596). In our model, the tax authority does

[10] Battaglini et al. (2020) consider a model of tax evasion by sole proprietorship taxpayers in Italy where taxpayers submit their returns after consulting tax accountants. There, the tax accountants aggregate information about the audits (or no audits) of their past clients, thereby transmitting information about the IRA audit probabilities. The tax authority, in turn, chooses audit probabilities optimally to maximize tax revenue net of audit costs. This interaction gives rise to self-selection of taxpayers into accountants with different attitudes about tax evasion and informational externalities arising from the tax accountant's communication. The information aggregation in our model occurs due to neighborhood information rather than any specialized information supplied by the intermediary accountants.

not make full, complex inferences through the lens of taxpayers' behavior that are based on their neighbors' profit realizations.

After the model description next, the equilibrium analysis is presented in Section 3 followed by Conclusions. An Appendix contains the proofs. Supplementary file contains codes for numerical simulations reported in Tables 1–4.[11]

2 The model

There are n self-employed individuals in an industry/business, interchangeably referred to as agents/entrepreneurs/taxpayers. Each agent i has an ability $\tau_i \in \{h, \ell\}$, with h denoting high ability and ℓ denoting low ability, where $\Pr(\tau_i = h) = q$, $0 < q < 1$. Agents experience a common market shock ϵ, favorable ($\epsilon = 1$) or unfavorable ($\epsilon = 0$), and $\Pr(\epsilon = 1) = \alpha$, $0 < \alpha < 1$. The actual draw of the common shock is unknown to all. Each agent draws his profit from a two-point support $y_i \in \{0, 1\}$ depending on his ability and the common market shock. Let

$$\rho_{\tau_i,\epsilon} \equiv \Pr(y_i = 1|\epsilon, \tau_i).$$

We assume that the following properties are satisfied:

$$\begin{gathered}\rho_{h,1} > \rho_{h,0}, \quad \rho_{\ell,1} > \rho_{\ell,0}, \quad \rho_{h,1} > \rho_{\ell,1}, \quad \text{and} \quad \rho_{h,0} > \rho_{\ell,0}, \\ \text{but} \quad \rho_{h,0} \quad \text{and} \quad \rho_{\ell,1} \quad \text{cannot be ranked.}\end{gathered} \tag{1}$$

From this partial ordering we can only conclude that the probability of a high-profit draw is increasing separately in ability and market shock.

Assumption 1 (Reliance on market). *The low-ability entrepreneur relies more on good market shock for his success than a high-ability entrepreneur:*

$$\frac{\rho_{\ell,1}}{\rho_{\ell,0}} > \frac{\rho_{h,1}}{\rho_{h,0}}. \tag{2}$$

[11] Numerical simulations have been carried out in Mathematica as well as in Excel for cross-checking.

The assumption is intuitive: when the market is buoyant even an unimpressive salesman can manage to sell high volumes (equivalently realize high profit), but when the market is down the salesman has to be more persuasive to sell high (i.e., to realize high profit).

■ **Information flow.** Each agent's ability is his private information, while the probabilities q, α and the technology $\rho_{\tau_i,\epsilon}$ are common knowledge among the agents and the tax authority. Also, agent i knows his own profit and the profit of his right-hand neighbor $i-1$ with the agents placed in a circle as in Fig. 1. The tax authority does not know the exact ordering of the agents although the unidirectional information structure is common knowledge. Nor do the agents know, beyond their right-hand neighbor, the positioning of the other $n-2$ agents. While this informational assumption is a simplification for reasons of tractability, it is also descriptive of how entrepreneurs may or may not know about each other engaged in the same trade through the word of mouth. We opted for lack of mutual knowledge to keep the complexity of taxpayer reporting strategies and equilibrium inference by the tax authority tractable.

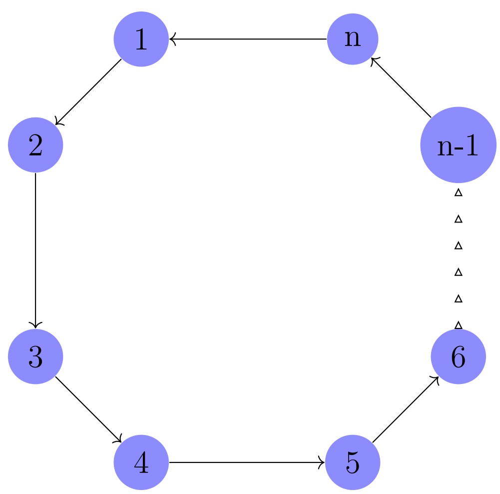

Figure 1: Each agent k knows the profit of his right-hand neighbor (facing away from the perimeter) $k-1$ clockwise, $k = 1, ..., n$ where $0 \equiv n$ for $k = 1$.

Each agent needs to report his profit to the tax authority. If the reported profit is 1, the agent needs to pay a tax $0 < t < 1$, and otherwise pay zero. If the agent is audited and found to have under-reported, he needs to pay back the tax t plus a fine $f > 0$. The tax authority is interested in collecting the tax and the fine, if any, and we assume that the true profit will be uncovered with certainty if auditing is conducted with a cost $c > 0$. Admittedly, the assumption of perfect discovery of the true profit on audit is a strong one but fairly standard in the tax enforcement literature. We believe that our qualitative findings should survive if the audit finds the truth with a reasonably high probability.

3 Tax returns data and auditing

The taxpayer i's submission depends on his own profit and type, and the profit of his neighbor $i-1$. Let $\hat{y}_i(y_i, y_{i-1}|\tau_i)$ denote his reporting strategy when his true profit is y_i, his neighbor's profit is y_{i-1} and his own type is τ_i. The taxpayer is risk-neutral, and thus compares his (expected) profit from truthful reporting with the one, inclusive of the fine, from under-reporting.

We will assume no exogenous commitment to an audit rule, unlike many papers on tax enforcement. The tax authority considers only the cost-benefit tradeoff and audits as long as the expected benefit exceeds the auditing cost. This is possible due to our assumption that there is no budget constraint for reasons discussed in the Introduction. In particular, we show the tax authority uses an ***audit cutoff*** strategy based on the total number of high submissions, as follows:

> *If the total number of high submissions is above or equal to a threshold number $\hat{m}$, the tax authority will audit all taxpayers who submitted low reports, and otherwise no one will be audited.*

Unlike in the literature where the audit cutoff is an income level, the above strategy does not require commitment power of the tax authority, and the exact value of the cutoff $\hat{m}$ will be generated through equilibrium analysis. For any particular taxpayer, though he knows the cutoff value, there is no way for him to know how many will submit a report of high profits, thus cannot "fine-tune" his reporting strategy. On the other hand, after receiving all the reports, the tax authority needs to guess the probability of under-reporting, and follow the cutoff strategy $\hat{m}$ to maximize the highest *residual* expected payoff (recovered tax plus fines less the auditing costs).

The interaction between the taxpayers and the tax authority (i.e., the auditor) is a sequential Bayesian game of social learning and coordination, where the taxpayers move first followed by the tax authority. We assume only one-sided asymmetric information, with information about the audit cost c being common knowledge. Without the market shock the game is a textbook auditor–single taxpayer game. The common shock links the taxpayers' strategies (submissions) through the neighborhood information. The auditor then formulates an auditing strategy based on aggregate submissions that convey information about the market shock. The equilibrium solution concept is *Perfect Bayesian Equilibrium*, in short, *PBE*.

For our analysis, below we propose a plausible reporting strategy and then derive the value of $\hat{m}$. Our ultimate aim is to show that the auditor's cutoff strategy with this derived $\hat{m}$ and our proposed taxpayers' reporting strategy can be supported as an equilibrium under appropriate conditions.

Posited equilibrium strategies for the taxpayers:

$$\begin{aligned} &\hat{y}_i(1,0|h) = 0, && \hat{y}_i(1,0|\ell) = 0, \\ &\hat{y}_i(1,1|h) = 0, && \hat{y}_i(1,1|\ell) = 1, \\ &\hat{y}_i(0,y_{i-1}|\tau) = 0, \quad \forall\tau. && \end{aligned} \tag{3}$$

Usually, low-ability taxpayer tends to attribute his high profit to favorable market shock while high-ability taxpayer attributes it to his high ability, implying that low-ability taxpayers report high profits more often. In fact, according to the posited equilibrium *only* the low-ability taxpayers will report high profits. But when a low-ability entrepreneur with a high profit observes his neighbor to have made a low profit, the differential profit experience dents his posterior about the market shock being favorable, prompting him to underreport.

From (3) we know that $\hat{y}_j = 0$ arises from one of the following three events:

- $y_j = 0,\ \tau_j \in \{h, \ell\}$
- $y_j = 1,\ \tau_j = h$
- $y_j = 1, \tau_j = \ell, y_{j-1} = 0.$

When tax submission strategies in (3) are viewed against the productivity technology satisfying the partial ordering in (1), it stands to reason that the tax department on receiving a high proportion of tax returns of $\hat{y} = 1$ would attach a high chance that the market shock has been favorable. This will be shown formally in Proposition 1. This prompts the tax authority to audit returns of $\hat{y} = 0$ if the fraction of $\hat{y} = 1$ exceeds a threshold value.

Now, suppose the tax authority receives m number of high reports out of a total of n reports, $m < n$. Then it follows that at least $m+1$ taxpayers realized high profits and those reporting high profits are all of low-ability types. Others who may have realized high profits but reported low profits are either of high or low abilities. In the absence of additional information on the structure of information flows, i.e., whose profit information an agent learns (equivalently the entire circular flow of information), any updating of beliefs about taxpayers' types and the market shock in a fully rational manner is a hard problem. In fact, it might not be plausible to assume that the tax authority will have such information to calibrate its auditing strategy. We therefore take the route of *bounded rational* updating by the tax authority based on the information that is revealed in m high-profit reports. Formal details we develop next.

Next, we are going to analyze the tax authority's and taxpayers' problems separately. In particular, we are going to construct tax authority's auditing incentive condition based on the proposed taxpayers' equilibrium reporting strategies, and look for conditions under which the taxpayers' reporting strategies are indeed incentive compatible given the cutoff auditing rule.

3.1 Tax authority's problem

Suppose the tax authority has received m high reports. With this information an auditor would audit a taxpayer reporting $\hat{y}_j = 0$ if and only if the expected profit from auditing is higher than the audit cost, i.e.,[12]

$$
\begin{aligned}
& \Pr(y_j = 1 | m \text{ high} \setminus \{j\})(t+f) \geq c, \\
\text{i.e.,} \quad & \Big[\Pr(\epsilon = 1 | m \text{ high} \setminus \{j\}) \cdot \Pr(y_j = 1 | m \text{ high} \setminus \{j\}, \epsilon = 1) + \\
& \quad \Pr(\epsilon = 0 | m \text{ high} \setminus \{j\}) \cdot \Pr(y_j = 1 | m \text{ high} \setminus \{j\}, \epsilon = 0)\Big](t+f) \geq c. \qquad (4)
\end{aligned}
$$

Define

$$
\begin{aligned}
\beta_m &\equiv \Pr(\epsilon = 1 | m \text{ high reports}), \\
\mu_{m,1} &\equiv \Pr(y_j = 1 | m \text{ high} \setminus \{j\}, \epsilon = 1), \\
\mu_{m,0} &\equiv \Pr(y_j = 1 | m \text{ high} \setminus \{j\}, \epsilon = 0).
\end{aligned}
$$

The posterior β_m is a critical piece of information for the auditing strategy and offers a first hint into what is going to be the basis of our main message in this paper about *data-driven auditing*; we will see later on that the tax authority will rely on the statistic m to decide on whether to spend the precious audit dollars to scrutinize a low submission.

Now the inequality (4) can be reduced to

$$
\big[\beta_m \mu_{m,1} + (1-\beta_m)\mu_{m,0}\big](t+f) \geq c. \qquad (5)
$$

Now, we are going to derive the expressions for β_m, $\mu_{m,1}$, and $\mu_{m,0}$ separately.

■ **Derivation for β_m.** We can write

$$
\beta_m = \frac{\Pr(m \text{ high reports} | \epsilon = 1) \times \Pr(\epsilon = 1)}{\Pr(m \text{ high reports} | \epsilon = 1) \times \Pr(\epsilon = 1) + \Pr(m \text{ high reports} | \epsilon = 0) \times \Pr(\epsilon = 0)}.
$$

[12]For the posterior calculation below, the information about the specific j not being in the list of m high reports is of no additional value.

Given our posited equilibrium taxpayer strategies shown in (3), let us first derive the probability that any generic taxpayer will report high profit given any market shock ϵ, as follows:

$$
\begin{aligned}
&\Pr(\hat{y}_k = 1|\epsilon = 1)\\
&= \Pr(y_k = 1, y_{k-1} = 1, \tau_k = \ell|\epsilon = 1)\\
&= \Pr(\tau_k = \ell) \cdot \Pr(y_k = 1, y_{k-1} = 1|\tau_k = \ell, \epsilon = 1)\\
&= \Pr(\tau_k = \ell) \cdot \Pr(y_k = 1|\tau_k = \ell, \epsilon = 1) \cdot \Pr(y_{k-1} = 1|\epsilon = 1)\\
&= \Pr(\tau_k = \ell) \cdot \Pr(y_k = 1|\tau_k = \ell, \epsilon = 1) \cdot \big[\Pr(y_{k-1} = 1, \tau_{k-1} = \ell|\epsilon = 1)\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \Pr(y_{k-1} = 1, \tau_{k-1} = h|\epsilon = 1)\big]\\
&= \Pr(\tau_k = \ell) \cdot \Pr(y_k = 1|\tau_k = \ell, \epsilon = 1) \cdot \big[\Pr(\tau_{k-1} = \ell)\Pr(y_{k-1} = 1|\tau_{k-1} = \ell, \epsilon = 1)\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \Pr(\tau_{k-1} = h)\Pr(y_{k-1} = 1|\tau_{k-1} = h, \epsilon = 1)\big]\\
&= (1-q)\rho_{\ell,1}[(1-q)\rho_{\ell,1} + q\rho_{h,1}] \equiv \theta,
\end{aligned}
$$

and

$$
\begin{aligned}
&\Pr(\hat{y}_k = 1|\epsilon = 0)\\
&= \Pr(\tau_k = \ell) \cdot \Pr(y_k = 1|\tau_k = \ell, \epsilon = 0) \cdot \big[\Pr(\tau_{k-1} = \ell)\Pr(y_{k-1} = 1|\tau_{k-1} = \ell, \epsilon = 0)\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \Pr(\tau_{k-1} = h)\Pr(y_{k-1} = 1|\tau_{k-1} = h, \epsilon = 0)\big]\\
&= (1-q)\rho_{\ell,0}[(1-q)\rho_{\ell,0} + q\rho_{h,0}] \equiv \theta'.
\end{aligned}
$$

So,

$$
\begin{aligned}
\Pr(m \text{ high reports}|\epsilon = 1) &= \binom{n}{m}\theta^m(1-\theta)^{n-m},\\
\Pr(m \text{ high reports}|\epsilon = 0) &= \binom{n}{m}\theta'^m(1-\theta')^{n-m},
\end{aligned}
$$

implying

$$\beta_m = \frac{\alpha \cdot \theta^m (1-\theta)^{n-m}}{\alpha \cdot \theta^m (1-\theta)^{n-m} + (1-\alpha)\theta'^m (1-\theta')^{n-m}}.$$

Proposition 1 (Volume statistic)**.** β_m*, expressed in terms of the model primitives, is increasing in* m*.*

This is an important result, and the intuition is simple: the higher the proportion of high reports which is akin to *volume statistic* in financial trading, the greater the likelihood of a favorable market shock. Such an inference is particularly appealing for the equilibrium to be constructed in which all high submissions will come from low-ability entrepreneurs. Focusing on such an equilibrium is realistic because only those who do not have confidence in their own abilities are more likely to consider their good fortune to be a matter of luck. That is, confidence (or the lack of it) leads to a corresponding attribution.

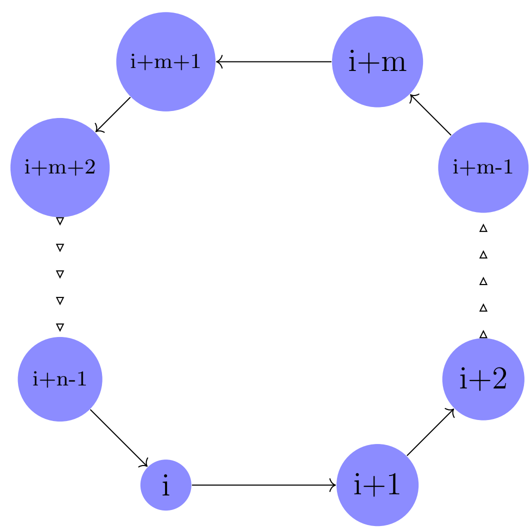

Figure 2: Taxpayers $i+1, ..., i+m$ all report $\hat{y} = 1$ and the rest $j = i+m+1, ..., i+n-1$ report $\hat{y}_j = 0$; $i-1 \equiv i+n-1$.

■ **Derivation for $\mu_{m,1}$ and $\mu_{m,0}$.** We now turn to estimate $\mu_{m,1}$ and $\mu_{m,0}$. This exercise, integral to determining the gross payoff from auditing a low-profit submission on the LHS of (5), can be quite complicated due to iteration in full Bayesian updating.

To understand the nature of the exercise, consider a very simple case where all m high-reports are *contagious*, as in Fig. 2. This would leave out $n-m$ reports that are all low. So when the tax authority faces an arbitrary taxpayer j who reported low, he has to conjecture where j might be located in the information chain in the arc starting from $i+m+1$ to i. If $j=i$, clearly $\Pr(y_j = 1|m \text{ high} \setminus \{j\}) = 1$; if $j = i+m+1$ then j might have underreported based on the information that $y_{i+m} = 1$; if $j = i+m+2$, then the tax authority can tell that j is likely acting on the twin possibilities that $y_{i+m+1} = 0$ and $y_{i+m+1} = 1$ whose probabilities the tax authority can estimate, using the posited equilibrium reporting strategies, that taxpayer $i+m+1$ had observed $y_{i+m} = 1$. And so on, down the chain. Further, given that j can be anywhere even in this simple two-arc reporting (all high reports contagious and all low reports are also contagious), the tax authority will have to assign a uniform probability, $1/(n-m)$, of j being in any of the $n-m$ low-report slots. Ultimately, a calculation of the probability that $y_j = 1$ given that $\hat{y}_j = 0$ will boil down to the calculations of all j's position-specific probabilities using iterative Bayesian updating according to the chain and then weighted by the density $1/(n-m)$. While, in principle, such updating and then averaging is theoretically possible, one cannot ignore the inevitability that there can be *gaps* between high reports that adds a stiff challenge for practical use of such a sophisticated calculation. So we propose a simple alternative approach explained below.

First observe that $\Pr(y_j = 1|m = n-1, \hat{y}_j = 0) = 1$, which is uninteresting. We will focus therefore on the case $m < n-1$. Given that the tax authority is unaware of the exact structure of neighborhood information flow, he will use a ***'bounded rationality'*** approach and ignore j's positions in any of the numerous possible contagiousness of high- and low-profit reporting. Instead, the tax authority follows a 'simple learning rule': he uses the information about the received m high reports to <u>only</u> update his belief about the common shock, and once the common shock is known, his updated belief about the taxpayers' evasion possibilities will be based only on their equilibrium reporting strategies. Thus, $\mu_{m,1}$ and $\mu_{m,0}$

will be approximated as

$$[\textbf{Bounded rationality}] \qquad \begin{aligned} \mu_{m,1} &\approx \Pr(y_j = 1 | \epsilon = 1) \equiv \mu_1, \\ \mu_{m,0} &\approx \Pr(y_j = 1 | \epsilon = 0) \equiv \mu_0, \end{aligned}$$

and the audit strategy condition (5) will be simplified as

$$\left[\beta_m \mu_1 + (1 - \beta_m)\mu_0\right](t + f) \geq c. \tag{6}$$

Now, we can express μ_1 as follows:

$$\mu_1 = \frac{A + B}{A + B + C},$$

where

$$\begin{aligned} A &= \Pr(y_j = 1, \tau_j = h | \epsilon = 1) \\ &= \Pr(y_j = 1 | \tau_j = h, \epsilon = 1) \cdot \Pr(\tau_j = h | \epsilon = 1) = \Pr(y_j = 1 | \tau_j = h, \epsilon = 1) \cdot \Pr(\tau_j = h) = \rho_{h,1} q, \\ B &= \Pr(y_j = 1, y_{j-1} = 0, \tau_j = \ell | \epsilon = 1) \\ &= \Pr(y_j = 1 | \tau_j = \ell, y_{j-1} = 0, \epsilon = 1) \times \Pr(\tau_j = \ell | y_{j-1} = 0, \epsilon = 1) \times \Pr(y_{j-1} = 0 | \epsilon = 1) \\ &= \Pr(y_j = 1 | \tau_j = \ell, \epsilon = 1) \times \Pr(\tau_j = \ell) \times \Pr(y_{j-1} = 0 | \epsilon = 1) \\ &= \rho_{\ell,1}(1 - q)[(1 - \rho_{h,1})q + (1 - \rho_{\ell,1})(1 - q)], \\ C &= \Pr(y_j = 0 | \epsilon = 1) = (1 - \rho_{h,1})q + (1 - \rho_{\ell,1})(1 - q). \end{aligned}$$

Similarly,

$$\mu_0 = \frac{A' + B'}{A' + B' + C'},$$

where

$$
\begin{aligned}
A' &= \Pr(y_j = 1, \tau_j = h | \epsilon = 0) \\
&= \Pr(y_j = 1 | \tau_j = h, \epsilon = 0) \cdot \Pr(\tau_j = h | \epsilon = 0) = \Pr(y_j = 1 | \tau_j = h, \epsilon = 0) \cdot \Pr(\tau_j = h) = \rho_{h,0} q, \\
B' &= \Pr(y_j = 1, y_{j-1} = 0, \tau_j = \ell | \epsilon = 0) \\
&= \Pr(y_j = 1 | \tau_j = \ell, y_{j-1} = 0, \epsilon = 0) \times \Pr(\tau_j = \ell | y_{j-1} = 0, \epsilon = 0) \times \Pr(y_{j-1} = 0 | \epsilon = 0) \\
&= \Pr(y_j = 1 | \tau_j = \ell, \epsilon = 0) \times \Pr(\tau_j = \ell) \times \Pr(y_{j-1} = 0 | \epsilon = 0) \\
&= \rho_{\ell,0}(1-q)[(1-\rho_{h,0})q + (1-\rho_{\ell,0})(1-q)], \\
C' &= \Pr(y_j = 0 | \epsilon = 0) = (1-\rho_{h,0})q + (1-\rho_{\ell,0})(1-q).
\end{aligned}
$$

After substituting the expressions of β, μ_1, and μ_0 into (6), we have the following intuitive results that will be key to determining the tax authority's equilibrium audit strategy in the audit-evasion game.

Lemma 1. **(i)** *$\mu_1 > \mu_0$;*

(ii) *The LHS of (6) is increasing in m.*

3.2 Taxpayer's problem

Next, consider the taxpayers' strategies. For a high-profit earner k, if he reports $\hat{y}_k = 1$, he needs to pay t; if he reports $\hat{y}_k = 0$, his expected payment is $\Pr(m \geq \hat{m} | y_k, y_{k-1}, \tau_k)(t + f)$ given the cutoff auditing strategy used by the tax authority. Thus we need to determine each taxpayer's belief about the number of high submissions assuming that all other taxpayers follow the strategies described in the posited equilibrium. In particular, for each $(y_k = 1, y_{k-1}, \tau_k)$ profile, we will derive conditions under which it is incentive compatible for him to follow the posited equilibrium reporting strategy. (We do not need to consider the incentives for low-profit earners as they will always report 0. Thus, there will be four incentive compatibility conditions.)

For taxpayer k, the probability of any particular taxpayer $j \neq k$ reporting 1 is

$$
\begin{aligned}
& \Pr(\hat{y}_j = 1 | y_k, y_{k-1}, \tau_k) \\
= & \begin{cases} \Pr(y_j = 1, y_{j-1} = 1, \tau_j = \ell | y_k, y_{k-1}, \tau_k), & \text{if } j \neq k-1 \\ \Pr(y_{j-1} = 1, \tau_j = \ell | y_k, y_{k-1}, \tau_k), & \text{if } j = k-1 \text{ and } y_{k-1} = 1 \\ 0, & \text{if } j = k-1 \text{ and } y_{k-1} = 0. \end{cases}
\end{aligned}
$$

For ease of calculation, we will focus only on the case of $j \neq k-1$ to update taxpayer k's belief and approximate the truthful reporting probability of his neighbor same as any other taxpayer. Therefore, we can write

$$
\begin{aligned}
& \Pr(\hat{y}_j = 1 | y_k, y_{k-1}, \tau_k) \\
= & \Pr(y_j = 1, y_{j-1} = 1, \tau_j = \ell | y_k, y_{k-1}, \tau_k) \\
= & \Pr(y_j = 1, y_{j-1} = 1 | \tau_j = \ell, y_k, y_{k-1}, \tau_k) \cdot \Pr(\tau_j = \ell) \\
= & \left[\Pr(y_j = 1, y_{j-1} = 1, \epsilon = 1 | \tau_j = \ell, y_k, y_{k-1}, \tau_k) + \Pr(y_j = 1, y_{j-1} = 1, \epsilon = 0 | \tau_j = \ell, y_k, y_{k-1}, \tau_k) \right] \cdot \Pr(\tau_j = \ell).
\end{aligned} \tag{7}
$$

Now, for any given $(\epsilon, y_k, y_{k-1}, \tau_k)$, the expression $\Pr(y_j = 1, y_{j-1} = 1, \epsilon | \tau_j = \ell, y_k, y_{k-1}, \tau_k)$ can be calculated as follows:

$$
\begin{aligned}
& \Pr(y_j = 1, y_{j-1} = 1, \epsilon | \tau_j = \ell, y_k, y_{k-1}, \tau_k) \\
& = \frac{\rho_{\ell,\epsilon} \rho_{\tau_k,\epsilon} [\rho_{h,\epsilon} q + \rho_{l,\epsilon}(1-q)] \cdot [\Pr(y_{k-1} | \tau_{k-1} = h, \epsilon) q + \Pr(y_{k-1} | \tau_{k-1} = \ell, \epsilon)(1-q)] \cdot \Pr(\epsilon)}{\left[\begin{array}{l} \rho_{\tau_k,1} [\Pr(y_{k-1} | \tau_{k-1} = h, \epsilon = 1) q + \Pr(y_{k-1} | \tau_{k-1} = \ell, \epsilon = 1)(1-q)] \alpha \\ \quad + \rho_{\tau_k,0} [\Pr(y_{k-1} | \tau_{k-1} = h, \epsilon = 0) q + \Pr(y_{k-1} | \tau_{k-1} = \ell, \epsilon = 0)(1-q)](1-\alpha) \end{array} \right]}.
\end{aligned} \tag{8}
$$

The derivation is included in the Appendix. For ease of calculations, define

$$
\begin{aligned}
T_1 &= \rho_{\ell,1}(1-q) + \rho_{h,1}q,\\
T_2 &= (1-\rho_{\ell,1})(1-q) + (1-\rho_{h,1})q,\\
T_3 &= \rho_{\ell,0}(1-q) + \rho_{h,0}q,\\
T_4 &= (1-\rho_{\ell,0})(1-q) + (1-\rho_{h,0})q.
\end{aligned}
$$

Note that $T_1 > T_3$ and $T_4 > T_2$ since $\rho_{\ell,1} > \rho_{\ell,0}$ and $\rho_{h,1} > \rho_{h,0}$.

We are going to analyze each $(y_k = 1, y_{k-1}, \tau_k)$ profile separately. The detailed analysis of **Case (1)** is presented next, and the analysis of **Cases (2)–(4)** is included in the Appendix, with only incentive compatibility conditions presented in the main text.

Case (1): $y_k = 1, y_{k-1} = 1, \tau_k = \ell$.

By substituting these values and $\epsilon = 1$ into equation (8), we obtain

$$
\begin{aligned}
&\Pr(y_j = 1, y_{j-1} = 1, \epsilon = 1 | \tau_j = \ell, y_k = 1, y_{k-1} = 1, \tau_k = \ell)\\
&= \frac{\left\{\rho_{\ell,1}\left[\rho_{\ell,1}(1-q) + \rho_{h,1}q\right]\right\}^2 \alpha}{\rho_{\ell,1}\left[\rho_{\ell,1}(1-q) + \rho_{h,1}q\right]\alpha + \rho_{\ell,0}\left[\rho_{\ell,0}(1-q) + \rho_{h,0}q\right](1-\alpha)}\\
&= \frac{(\rho_{\ell,1}T_1)^2\alpha}{\rho_{\ell,1}T_1\alpha + \rho_{\ell,0}T_3(1-\alpha)},
\end{aligned}
$$

and

$$
\begin{aligned}
&\Pr(y_j = 1, y_{j-1} = 1, \epsilon = 0 | \tau_j = \ell, y_k = 1, y_{k-1} = 1, \tau_k = \ell)\\
&= \frac{\left\{\rho_{\ell,0}\left[\rho_{\ell,0}(1-q) + \rho_{h,0}q\right]\right\}^2 (1-\alpha)}{\rho_{\ell,1}\left[\rho_{\ell,1}(1-q) + \rho_{h,1}q\right]\alpha + \rho_{\ell,0}\left[\rho_{\ell,0}(1-q) + \rho_{h,0}q\right](1-\alpha)}\\
&= \frac{(\rho_{\ell,0}T_3)^2(1-\alpha)}{\rho_{\ell,1}T_1\alpha + \rho_{\ell,0}T_3(1-\alpha)}.
\end{aligned}
$$

Thus from (7), define

$$\eta_{1,1,\ell} \equiv \Pr(\hat{y}_j = 1 | y_k = 1, y_{k-1} = 1, \tau_k = \ell) = \frac{(\rho_{\ell,1}T_1)^2\alpha + (\rho_{\ell,0}T_3)^2(1-\alpha)}{\rho_{\ell,1}T_1\alpha + \rho_{\ell,0}T_3(1-\alpha)} \times (1-q).$$

Similarly, in the Appendix we derive the following beliefs:

$$\eta_{1,0,\ell} \equiv \Pr(\hat{y}_j = 1 \big| y_k = 1, y_{k-1} = 0, \tau_k = \ell) = \frac{\rho_{\ell,1}^2 T_1 \cdot T_2\alpha + \rho_{\ell,0}^2 T_3 \cdot T_4(1-\alpha)}{\rho_{\ell,1}T_2\alpha + \rho_{\ell,0}T_4(1-\alpha)} \times (1-q),$$

$$\eta_{1,1,h} \equiv \Pr(\hat{y}_j = 1 \big| y_k = 1, y_{k-1} = 1, \tau_k = h) = \frac{\rho_{\ell,1}\rho_{h,1}T_1^2\alpha + \rho_{\ell,0}\rho_{h,0}T_3^2(1-\alpha)}{\rho_{h,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)} \times (1-q),$$

$$\eta_{1,0,h} \equiv \Pr(\hat{y}_j = 1 \big| y_k = 1, y_{k-1} = 0, \tau_k = h) = \frac{\rho_{\ell,1}\rho_{h,1}T_1 \cdot T_2\alpha + \rho_{\ell,0}\rho_{h,0}T_3 \cdot T_4(1-\alpha)}{\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)} \times (1-q).$$

Now, we calculate the probability of there being more than $\hat{m}$ such taxpayers:

$$\begin{aligned}
&\Pr(m \geq \hat{m} | y_k = 1, y_{k-1} = 1, \tau_k = \ell) \\
&= 1 - \Pr(m < \hat{m} | y_k = 1, y_{k-1} = 1, \tau_k = \ell) \\
&= 1 - \sum_{i=0}^{\hat{m}-1} \frac{n!}{i!(n-i)!} \eta_{1,1,\ell}^i (1-\eta_{1,1,\ell})^{n-i}.
\end{aligned}$$

Therefore, truthful reporting by agent k with $(y_k = 1, y_{k-1} = 1, \tau_k = \ell)$ requires:

$$\begin{aligned}
&\Pr(m \geq \hat{m} | y_k = 1, y_{k-1} = 1, \tau_k = \ell)(t+f) \geq t \\
\text{i.e.,}\quad &\left[1 - \sum_{i=0}^{\hat{m}-1} \frac{n!}{i!(n-i)!} \eta_{1,1,\ell}^i (1-\eta_{1,1,\ell})^{n-i}\right](t+f) \geq t.
\end{aligned}$$

Using similar methods, we derive in the Appendix three other incentive compatibility conditions from **Cases (2)–(4)** as follows:

$$\left[1-\sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,0,\ell}^{i}(1-\eta_{1,0,\ell})^{n-i}\right](t+f)\le t,$$
$$\left[1-\sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,1,h}^{i}(1-\eta_{1,1,h})^{n-i}\right](t+f)\le t,$$
$$\left[1-\sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,0,h}^{i}(1-\eta_{1,0,h})^{n-i}\right](t+f)\le t.$$

Note that the above analysis is well-defined only for $\hat{m}>0$. When $\hat{m}=0$, no taxpayer has an incentive to underreport because the evasion will be detected with probability 1. But given truthful reporting by the taxpayers, the tax authority's best response should be to not audit at all. Thus, there is no pure strategy equilibrium associated with $\hat{m}=0$.

Definition 1. *For any $\hat{m}\ge 1$, define*

$$\frac{f}{t+f}\Big|_{\min}(\hat{m})\equiv\sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,1,\ell}^{i}(1-\eta_{1,1,\ell})^{n-i},$$

and $$\frac{f}{t+f}\Big|_{\max}(\hat{m})\equiv\min\Bigg\{\sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,0,\ell}^{i}(1-\eta_{1,0,\ell})^{n-i},$$
$$\sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,1,h}^{i}(1-\eta_{1,1,h})^{n-i},\ \sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,0,h}^{i}(1-\eta_{1,0,h})^{n-i}\Bigg\}.$$

Thus, the incentive compatibility conditions can be simplified as

$$\frac{f}{t+f}\Big|_{\min}(\hat{m})\le\frac{f}{t+f}\le\frac{f}{t+f}\Big|_{\max}(\hat{m}).\tag{9}$$

3.3 Endogenous audit equilibrium

In order to derive the optimal cutoff value $\hat{m}$, we first arrive at the following ordering involving $\eta_{y_k,y_{k-1},\tau_k}$.

Lemma 2 (Ordering)**.** *Under Assumption 1, we have the following orderings:*

1. $\eta_{1,0,h} < \eta_{1,0,\ell} < \eta_{1,1,\ell}$;

2. $\eta_{1,0,h} < \eta_{1,1,h} < \eta_{1,1,\ell}$.

The intuition for $\eta_{1,0,\ell} < \eta_{1,1,\ell}$ is that an evidence of $y_{k-1} = 0$ as opposed to $y_{k-1} = 1$ will make agent k more pessimistic about the possibility that $\epsilon = 1$. In turn, the likelihood of $y_{j-1} = 1$ and hence high-profit report by agent j will be less. This will lead to a lower expected value of m, that we are going to reason, would prompt a low-ability agent, who observes his right-hand neighbor's profit to be low, to take a chance and underreport when his own profit is high. The intuition for $\eta_{1,0,h} < \eta_{1,1,h}$ is similar. The intuition for $\eta_{1,0,h} < \eta_{1,0,\ell}$ again relies on the ℓ-ability entrepreneur placing higher odds on the market shock being favorable than an h-ability entrepreneur and accordingly makes a higher projection that a representative taxpayer is of type $(1, 1, \ell)$. Finally, the reason $\eta_{1,0,\ell}$ and $\eta_{1,1,h}$ cannot be ranked unambiguously is that the difference in abilities and the difference in neighbors' profits pull the belief about the market shock in opposite directions.

Note that $\eta_{1,1,\ell}$ is the highest in both orderings, implying a low-ability taxpayer (with high profit) who observes his neighbor to have high profit places the highest odds that any representative (high profit) taxpayer would make truthful declaration. The intuition is that he places the highest odds (compared to the other types) on the market shock being favorable that in turn would impact positively on others' profits. This is also the reason why any equilibrium with truthful declaration of high profits must include the $(1, 1, \ell)$ types.[13]

As shown in Lemma 2, it is possible to rank the values of some of the beliefs $\eta_{y_k,y_{k-1},\tau_k}$, associated with different taxpayer profiles, but a clear ranking of the binomial summations involving $\eta_{y_k,y_{k-1},\tau_k}$ is not possible. In particular, the rate of increase of the four summations with respect to $\hat{m}$ is not order-preserving for all $\hat{m}$ from 0 to n. For example, when $\hat{m}$ is small, it might be the case that the summation involving $\eta_{1,0,\ell}$ is larger than the summation involving $\eta_{1,0,h}$, but this order is reversed for a larger value of $\hat{m}$. The need to work around this issue leads us to Definition 2 below, and is also a key reason why our main result

[13] For this paper, we restrict to equilibrium where $(1, 1, \ell)$ are the only truthful types.

(Proposition 2) is written in terms of sufficient conditions.

We are now ready to present our central results.

Definition 2. *Let* m^* *be the integer such that*

$$m^* \equiv \max\{m \mid m < \eta_{1,0,h} n + 1\}.$$

Lemma 3. *Under Assumption 1, when* $1 \le \hat{m} \le m^*$*, we have* $\frac{f}{t+f}\big|_{\min}(\hat{m}) < \frac{f}{t+f}\big|_{\max}(\hat{m})$.

Let us recall condition (6), the simplified version of (5), for the auditor to audit:

$$\left[\beta_m \mu_1 + (1-\beta_m)\mu_0\right](t+f) \ge c.$$

The LHS involves parameters ρ's, q, α and $t+f$ but not c. For us to specify the audit strategy fully, we need to divide the cost range into subintervals. With that in mind, let us define moving threshold values of high reports $\hat{m}$ with $m \ge \hat{m}$ triggering audits for varying cost ranges:

Definition 3 (Cost ranges for audits)**.** *Let*

$$\begin{aligned}
&c_0 \ \textit{ be such that } \ c_0 \equiv \left[\beta_0 \mu_1 + (1-\beta_0)\mu_0\right](t+f),\\
&c_1 \ \textit{ be such that } \ c_1 \equiv \left[\beta_1 \mu_1 + (1-\beta_1)\mu_0\right](t+f),\\
&c_2 \ \textit{ be such that } \ c_2 \equiv \left[\beta_2 \mu_1 + (1-\beta_2)\mu_0\right](t+f),\\
&\dots\\
&c_{m^*} \ \textit{ be such that } \ c_{m^*} \equiv \left[\beta_{m^*} \mu_1 + (1-\beta_{m^*})\mu_0\right](t+f).
\end{aligned}$$

Clearly, by applying Lemma 1 we have:

$$0 < c_0 < c_1 < c_2 < c_3 < \ldots < c_{m^*}.$$

Definition 4 (Audit strategy)**.** *Fix all parameters except the audit cost* c*. The following cutoff audit strategy based on the audit cost will be later shown to be part of our audit-evasion equilibrium:*

- *for* $c \in (c_0, c_1]$*, the tax authority will audit for all* $m \geq 1$ *and not audit if* $m = 0$*, that is,* $\hat{m} = 1$*;*
- *for* $c \in (c_1, c_2]$*, the tax authority will audit for all* $m \geq 2$ *and not audit if* $m < 2$*, that is,* $\hat{m} = 2$*;*

  ...;
- *for* $c \in (c_{m^*-1}, c_{m^*}]$*, the tax authority will audit for all* $m \geq m^*$ *and not audit if* $m < m^*$*, that is,* $\hat{m} = m^*$*.*

As to be expected, the cutoff $\hat{m}$ is increasing in the cost of audit. Because $\hat{m}$ can only be integers, any particular value of $\hat{m}$ remains applicable for a cost interval. Each time cost moves up from one interval to the adjoint interval, $\hat{m}$ increases by 1, thus taking the form of a step-ladder.

In a way, given t and f the audit cost c uniquely pins down the cutoff $\hat{m}$ for the posited taxpayer reporting. The construction of the equilibrium of the tax evasion game then comes down to specifying conditions such that the taxpayer strategies are incentive compatible for the $\hat{m}$. In our main result below, we put together a set of sufficient conditions to guarantee an equilibrium exhibiting our posited profit reporting and audit behaviors. The conditions require some flexibility with regard to (t, f) and the permissible ranges of c.

Proposition 2 (High-submission trigger)**.** *Suppose Assumption 1 holds. Consider any* $1 \leq \hat{m} \leq m^*$*.*

For any $c \in (c_{\hat{m}-1}, c_{\hat{m}}]$ *as in Definition 3, provided that* $\frac{f}{t+f}\big|_{\min}(\hat{m}) \leq \frac{f}{t+f} \leq \frac{f}{t+f}\big|_{\max}(\hat{m})$*, the following strategies constitute a Perfect Bayesian equilibrium of the tax evasion–audit game:*

(i) the tax authority will adopt the cutoff strategy of auditing all low-profit reports if $m \geq \hat{m}$*, and not audit otherwise;*

(ii) the taxpayers will choose evasion strategies according to (3)*.*

The implications of neighborhood information can be summarized as follows:

(i) It confirms or confounds a low-ability, high-profit achiever's assessment of the market shock and thus determines whether he should risk underreporting. Usually such a person would attribute high profit to a favorable market shock and is thus reluctant to underreport because he anticipates others in his situation are also likely to experience high profits and report truthfully. If he observes that his neighbor also has achieved high profit, then his posterior of a favorable market shock tends to get a further boost deterring him from evasion. However, if the neighbor has a low profit, then the posterior of a favorable market shock gets dampened and induces him to evade by underreporting, because he expects not many high submissions. The high type, high profit achievers think that their high profits are more due to their high abilities and underplays the role of a favorable market shock. Therefore this group always underreports.

(ii) The diffused information about the market shock gets transmitted, although imperfectly, to the tax authority in the form of the number of high submissions. For this number exceeding a threshold $\hat{m}$, the tax authority finds auditing worthwhile, whereas if the number of high submissions falls below the threshold the expectation of a favorable market shock is dampened and thus auditing is found unprofitable.

■ **Numerical illustration.** Proposition 2 offers a set of sufficient conditions for the equilibrium to exist. The construction of such an equilibrium can proceed as follows:

(i) Fix the parameters $(q, \alpha, \rho_{h,1}, \rho_{h,0}, \rho_{\ell,0}, \rho_{\ell,1}, n)$ at suitable values satisfying the relevant restrictions: $0 < q < 1$, $0 < \alpha < 1$, $0 < \rho_{h,1}, \rho_{h,0}, \rho_{\ell,0}, \rho_{\ell,1} < 1$, $\rho_{h,1} > \rho_{h,0}$, $\rho_{\ell,1} > \rho_{\ell,0}$, $\rho_{h,1} > \rho_{\ell,1}$, $\rho_{h,0} > \rho_{\ell,0}$, and $\frac{\rho_{\ell,1}}{\rho_{\ell,0}} > \frac{\rho_{h,1}}{\rho_{h,0}}$.

(ii) Determine $\eta_{1,0,h}$, and then $m^* = \eta_{1,0,h} n + 1$.

(iii) Choose any $\hat{m} \leq m^*$.

(iv) Calculate the four summation series in Definition 1 to determine the two bounds $\frac{f}{t+f}\big|_{\min}(\hat{m})$ and $\frac{f}{t+f}\big|_{\max}(\hat{m})$ in (9) for the taxpayers' incentive compatibility.

(v) Choose (t, f) so that $\frac{f}{t+f}$ remains within the bounds derived in step (iv).

(vi) Then calculate $\big[\beta_{\hat{m}-1}\mu_1 + (1-\beta_{\hat{m}-1})\mu_0\big](t+f)$ and $\big[\beta_{\hat{m}}\mu_1 + (1-\beta_{\hat{m}})\mu_0\big](t+f)$ (refer the auditor's incentive compatibility condition (6)) to obtain the lower and upper bounds for c. The range of c values thus constructed, for the chosen (t, f), would then satisfy both the taxpayers' ICs (9) and the auditor's ICs (6) for equilibrium $\hat{m}$.

Applying the above method, we construct the examples reported in Tables 1–4.

Table 1: Equilibrium,[a] fixing $(q, \alpha, \rho_{h,1}, \rho_{h,0}, \rho_{\ell,1}, \rho_{\ell,0}, t, f, n) = (0.3, 0.5, 0.8, 0.5, 0.3, 0.1, 0.4, 0.011, 200)$

	$m^* = 12$	$\frac{f}{t+f}\big\|_{\min} \leq \frac{f}{t+f} = 0.027 \leq \frac{f}{t+f}\big\|_{\max}$	Cost range
1.	$\hat{m} = 9$	$[0.012118, 0.0289974]$	$0.0985318 < c < 0.129633$
2.	$\hat{m} = 10$	$[0.0258064, 0.0566261]$	$0.129633 < c < 0.153986$

[a] Notice that an overlap in the ranges of $[\frac{f}{t+f}\big|_{\min}, \frac{f}{t+f}\big|_{\max}]$ for $\hat{m} = 9$ and $\hat{m} = 10$ allows us to choose a common $\frac{f}{t+f} = 0.027$ and a corresponding pair $(t, f) = (0.4, 0.011)$. This gives rise to the two cost ranges without any break, so we have a step-ladder relation between c and the cutoff $\hat{m}$.

Table 2: Equilibrium,[a] fixing $(q, \alpha, \rho_{h,1}, \rho_{h,0}, \rho_{\ell,1}, \rho_{\ell,0}, t, f, n) = (0.3, 0.5, 0.8, 0.5, 0.3, 0.1, 0.4, 0.0232, 200)$

	$m^* = 12$	$\frac{f}{t+f}\big\|_{\min} \leq \frac{f}{t+f} = 0.055 \leq \frac{f}{t+f}\big\|_{\max}$	Cost range
1.	$\hat{m} = 10$	$[0.0258064, 0.0566261]$	$0.133481 < c < 0.158557$
2.	$\hat{m} = 11$	$[0.0496002, 0.100023]$	$0.158557 < c < 0.164908$

[a] For the choice of a common $\frac{f}{t+f} = 0.055$ and $(t, f) = (0.4, 0.0232)$, we generate the two cost ranges without a break. There is a step-ladder relation between c and $\hat{m}$.

As one can see, there is a maximum of two $\hat{m}$ values that can be sustained with the step-ladder property for any (t, f) pair. This is so because the interval $[\frac{f}{t+f}\big|_{\min}, \frac{f}{t+f}\big|_{\max}]$ for

Table 3: Equilibrium,[a] fixing $(q, \alpha, \rho_{h,1}, \rho_{h,0}, \rho_{\ell,1}, \rho_{\ell,0}, t, f, n) = (0.3, 0.5, 0.8, 0.5, 0.3, 0.1, 0.4, 0.0444, 200)$

$m^* = 12$		$\frac{f}{t+f}\Big\rvert_{\min} \leq \frac{f}{t+f} = 0.1 \leq \frac{f}{t+f}\Big\rvert_{\max}$	Cost range
1.	$\hat{m} = 11$	$[0.0496002, 0.100023]$	$0.1665 < c < 0.17317$
2.	$\hat{m} = 12$	$[0.0870028, 0.161665]$	$0.17317 < c < 0.174274$

[a] For the choice of a common $\frac{f}{t+f} = 0.1$ and $(t, f) = (0.4, 0.0444)$, we generate the two cost ranges without a break. Again, there is a step-ladder relation between c and $\hat{m}$.

the taxpayers' incentive compatibility conditions keeps shifting to the right as $\hat{m}$ increases. And for a common $\frac{f}{t+f}$ to fit in within these bounds for more than one $\hat{m}$, the upperbound $\frac{f}{t+f}\big\rvert_{\max}(\hat{m}-1)$ must be below the lowerbound $\frac{f}{t+f}\big\rvert_{\min}(\hat{m})$ which is possible only for two consecutive values of $\hat{m}$. Thus, while the audit strategy in Definition 4 displaying the step-ladder characteristic is written for all $\hat{m} \leq m^*$, the taxpayers' incentive compatibility conditions (9) restrict equilibrium $\hat{m}$ to only two values. In our illustrative examples, we gave ourselves the freedom to choose (t, f) flexibly. But in actual enforcement (t, f) is determined at the fiscal division of the government. So, given any arbitrary c together with (t, f), the equilibrium in Proposition 2 may or may not materialize.

The above discussion should not diminish the value of our result, in particular the importance of enforcement applying the simple endogenous cutoff audit rule. It is worth emphasizing that we have considered only one type of equilibrium where only $(1, 1, \ell)$ type truthfully reports high profit. There could be other equilibria, for example both $(1, 1, \ell)$ and $(1, 1, h)$ types declaring their high profits truthfully. The reason $(1, 1, h)$ type may also declare truthfully is that a high-ability entrepreneur cannot be so confident that his high profit is due to high ability and not predominantly because the market shock could be favorable. It is possible that the neighbor's high profit is largely because of the favorable market shock if the neighbor happens to be of low ability. If q is not sufficiently high so that the probability of the neighbor being of low ability is reasonably large, $(1, 1, h)$ type would incur a risk by underreporting that he can ill afford.[14] Or it could be that $\rho_{h,0}$ is not high enough, so own

[14]In fact, for Table 1, if we lower the value of q from 0.3 to 0.2 while keeping other parameters unchanged,

high profit cannot be attributed to, predominantly, the entrepreneur's high ability. For this alternative equilibrium with two types reporting truthfully, again an increase in the statistic of the proportion of high submission would steer the balance of probability towards a favorable market shock. And when this statistic crosses a threshold value, the tax authority would be inclined to audit all low submissions.

We abstained from looking at this alternative equilibrium for two reasons. One, the logic and the method of equilibrium construction would be very similar. Second, the derivation of various posteriors and the ICs would be much more involved. But it is worth noting an interesting tradeoff of this second equilibrium. Because both high- and low-ability entrepreneurs report their high profits (as long as neighbor's profit is also high), the tax authority would have much richer information to evaluate the likelihood of the market shock: profit involves both skill and market shock as the two contributory factors. So audit decisions will be more accurate, on average.[15] The downside is that now there are fewer evaders, so the per-dollar return on audits will be less. The exact calculus cannot be ascertained without looking at detailed derivations, but we view this aspect of second-order importance for us not to undertake this exercise.

■ **Comparative statics.** We now address the standard comparative statics question of the effect of changes in some of the model parameters on the cutoff $\hat{m}$. Note that if in the initial equilibrium none of the ICs are binding, small change in any of the parameters leaves the equilibrium behavior unchanged. Assuming the ICs are non-binding, we will therefore consider the effect of "large" changes. Also, for lack of an explicit, simple-to-use solution for $\hat{m}$, our discussion will provide only an intuitive guidance. But one should also exercise caution because we do not offer an exhaustive characterization of the various equilibria

when $\hat{m} = 9$, the upperbound of $\frac{f}{t+f}$ decreases to 0.2281328 which is smaller than 0.027, and it is exactly due to the breakdown of the incentive compatibility condition for the $(1, 1, h)$ type. In other words, if there is an increase in the prior belief about the low-ability type $(1 - q)$, $(1, 1, h)$ will be the most reluctant group to underreport besides the $(1, 1, \ell)$ type.

[15]The intuition is similar to that of a market maker in asset trading. If the market maker has the information that the proportion who are actively trading are informed traders, incoming large buy orders would suggest that the traders on average have favorable information about the asset's value. The market maker should then increase the price.

possible.

(1) As the tax rate t increases, one can anticipate that evasion should increase. But in our equilibrium the only type that does not evade is $\tau_{\text{non-evader}} = (1, 1, \ell)$. So starting from an initial equilibrium, as t increases we need to consider type $\tau_{\text{non-evader}}$'s behavior. Suppose there is a $\bar{t}$ beyond which all non-evaders switch to evasion. Then in response, unless the initial $\hat{m}$ is very small, the cutoff $\hat{m}$ must fall. The only two places where t matters in the equilibrium construction are (i) in determining the cost grids $0 < c_0 < c_1 < c_2 < c_3 < \ldots < c_{m^*}$ for audits (Definition 3), and (ii) in defining the range $\frac{f}{t+f}\big|_{\min}(\hat{m}) \leq \frac{f}{t+f} \leq \frac{f}{t+f}\big|_{\max}(\hat{m})$ to make taxpayers' strategies incentive compatible. Now it is easy to see that an increase in t would shift the cost grids uniformly upwards, and thus likely shift any given c to the lower cost grid lowering $\hat{m}$.[16] And this lowered $\hat{m}$ would lower both the bounds in the taxpayers' ICs (see (ii) above, and Definition 1 and (9)) so that the lowered value of $\frac{f}{t+f}$ will be compatible with the ICs for the smaller $\hat{m}$.[17]

(2) As fine for evasion f increases, incentive for evasion should drop. Casual intuition suggests that perhaps the tax authority should monitor less intensively. This would be equivalent to $\hat{m}$ increasing. However, so long as the taxpayers' equilibrium strategies do not change, it is easy to see that the tax authority should instead audit more intensively, i.e., $\hat{m}$ should fall. The simple reason is that the expected benefit from auditing has gone up. This possibility is illustrated in Table 4: an increase in f from $f = 0.021$ to $f = 0.023$ lowers $\hat{m}$ from $\hat{m} = 11$ to $\hat{m} = 10$ for the cost range $0.157732 < c < 0.158482$. Of course if we were to consider other types of equilibria, an increase in f might lead to greater deterrence; for example $(1, 1, h)$ switches from non-truthful

[16] For example, in Table 2 a change of t from $t = 0.4$ to $t = 0.425$ holding $f = 0.0232$ unchanged (and $\frac{f}{t+f} = 0.052$, satisfying taxpayers' ICs) moves the cost grids upwards to $0.141366 < c < 0.167923$ for $\hat{m} = 10$ and to $0.167923 < c < 0.17465$ for $\hat{m} = 11$. Thus, if initially (i.e., when $t = 0.4$) $c = 0.167$ so that the corresponding $\hat{m} = 11$, now with the increase in t to $t = 0.425$, the same $c = 0.167$ is associated with $\hat{m} = 10$.

[17] In Table 2, moving from $\hat{m} = 11$ to $\hat{m} = 10$ meant $\frac{f}{t+f} = 0.052 \in [0.0258064, 0.0566261] \cap [0.0496002, 0.100023]$, thus satisfying taxpayers' ICs for both $\hat{m} = 10$ and $\hat{m} = 11$.

reporting to truthful reporting, resulting in an increase in $\hat{m}$ that reflects the familiar Beckerian type substitution between punishment and monitoring.

Table 4: Equilibrium, fixing $(q, \alpha, \rho_{h,1}, \rho_{h,0}, \rho_{\ell,1}, \rho_{\ell,0}, t, n) = (0.3, 0.5, 0.8, 0.5, 0.3, 0.1, 0.4, 200)$

	$m^* = 12$	$[\frac{f}{t+f}\big\|_{min}, \frac{f}{t+f}\big\|_{max}]$	f	$\frac{f}{t+f}$	Cost range
1.	$\hat{m} = 10$	$[0.0258064, 0.0566261]$	0.023	0.05437	$0.133418 < c < 0.158482$
2.	$\hat{m} = 11$	$[0.0496002, 0.100023]$	0.021	0.04988	$0.157732 < c < 0.164051$

(3) As c increases, monitoring becomes more costly and thus less intensive. That is, $\hat{m}$ should increase. This intuition is easy to verify: the only IC we need to consider is that of the auditor; going to Definition 3, the effect is that the increased c moves the auditor into the next higher grid from $(c_{\hat{m}-1}, c_{\hat{m}}]$ to $(c_{\hat{m}}, c_{\hat{m}+1}]$. This prediction is also verified in the three examples, Tables 1–3.

(4) Bringing in budget constraint and making it binding poses another comparative static question. Currently if $m \geq \hat{m}$, all $(n - m)$ low submissions will be audited at a total cost of $(n - m)c$. Our analysis without any budget consideration can also be interpreted as one where the tax authority has budget $B \geq (n - \hat{m})c$. A lowering of the budget to $B < (n - \hat{m})c$ would suggest, the $(1, 1, \ell)$ type taxpayers would now consider the possibility of being audited with less than probability 1. This might induce this group to adopt a mixed strategy and report with probability less than 1. This, in turn, results in a higher m underreporting on average. This should lower the cutoff $\hat{m}$. We abstain from analyzing this heuristic formally, because then we have to solve the coordination game of reporting (equivalently, solve for a mixed strategy reporting equilibrium), recalculate the various posteriors and derive the ICs afresh.

4 Variations, an extension and a limitation

Using the bounded rational learning formulation, an approach adopted in many other learning models of economic applications discussed in the Introduction, we are able to offer an intuitive and easy-to-implement guidance on optimal tax audits of self-employed entrepreneurs. For tax enforcement, this group poses the biggest challenge as evidenced in the policy focus on estimates of *gross tax gap*. Our analysis requires no commitment by the tax authority to an exogenous audit rule. Instead, optimal audits arise endogenously in equilibrium of the auditor–taxpayers game. A large previous literature on tax enforcement relied on commitment to an exogenous audit rule. Our analysis should be seen as advancing this literature in an important direction.

We conclude by discussing (i) the choice of the particular equilibrium over any other candidate, (ii) the role of any additional information at tax authority's disposal, (iii) possible extensions, and (iv) one limitation of our model.

■ **Choice of equilibrium.** Our choice of equilibrium is dictated by the message we wanted to convey. In the alternative structure where there is no neighborhood information, taxpayer strategies can only depend on their private type and their own information.

In the second environment, one can plausibly conjecture that an equilibrium is likely to involve all low types with high profit reporting truthfully (because they attribute high profit to good fortune, i.e., favorable market shock) and all high types with high profit underreporting (because they attribute high profit to their skills).

In the first environment, the focus of this paper, there can be other equilibria. For example, some of the other low types and high types can reveal truthfully. If $(1, 0, \ell)$ type reports truthfully, it becomes identical to the second environment behavior. If $(1, 1, h)$ reports truthfully (because he takes neighbor's high profit to be an indication that the market shock has been favorable with a high probability), then once again the result would confirm that information about the neighbor can induce you to give more weight to the market shock in your decision making. But already we include this type of influence in the low type's decision making.

The contrast between the first and the second environment can be further seen in an interesting feature, that of neighborhood information through which the market shock related information is brought into play in enforcement. This we are able to do by considering one class of equilibrium rather than having to analyze all possible equilibria. Considering other equilibria will bring in more of the same – the involved nature of the computations (ICs) – without adding any different insight.

■ **Tax authority's information.** We assumed the tax authority to have no information about the market shock for specific self-employment activities. For some sectors, the tax authority may obtain imperfect signals of the market shock. For instance, the information about a local boom in the demand for new residential constructions in some regions may be known, albeit with some noise. Our analysis can be extended to situations where the tax authority observes an imperfect signal about the market shock.

Suppose the signal is one of a favorable market shock but the proportion of tax submissions of high profits is relatively small. The favorable signal would suggest a relatively low cutoff $\hat{\mathfrak{m}}$, but the unexpectedly low number of high profit returns may prompt the tax authority to revise its cutoff $\hat{\mathfrak{m}}$. This revision of beliefs makes the taxpayer–auditor game more subtle. In contrast to our earlier analysis of taxpayer strategies, should the entrepreneurs behave any differently now? They know that if a proportion of them, i.e. (low-ability, high profit, high profit) types, nontruthfully submit low returns, the cutoff $\hat{\mathfrak{m}}$ itself will come down and the auditor may not audit low returns. The problem then turns into a more complex estimation of the cutoff $\hat{\mathfrak{m}}$ that requires a different analysis.

■ **Sampled auditing.** Our analysis assumed one-shot auditing: either all low-profit submissions are audited or none will be audited. An alternative approach would be to do audits in stages: learn about the environment by first sampling a small fraction of low-profit reports, and then recalibrate the expected cost–benefit numbers and revise the audit strategy.

This changes the taxpayer–auditor game substantially that requires a more involved analysis. This approach is similar to polling before the elections and running targeted campaigns in marginal districts. Our model should be seen as a first step to this and any other learning based auditing.

■ **A limitation: Unpredictability of auditing.** The following example shows why auditing can also be very unpredictable, as the equilibrium reporting is sensitive to the ordering of the agents. The agents' information changes with a change in the order, and so do their actual reporting. So the construction of an equilibrium has to be done carefully.

This example suggests, roughly, that for any given profile of $\{\tau_k, y_k, y_{k-1}\}$, the ordering that would maximize high submissions is when all high profit earners are positioned successively in the circle, whereas an ordering that opens 'holes' by sandwiching low profit earners between high profit earners as in the upper-right picture of Fig. 3 would minimize high submissions. So we need to find a way to estimate the *best* and *worst* values of m, and see whether the tax authority can glean these values from the actual number of high submissions. It is going to be a hard problem. But currently, our equilibrium as presented in Proposition 2 is *independent* of the various ordering of the agents and thus likely to be intermediate between the best and the worst estimates.

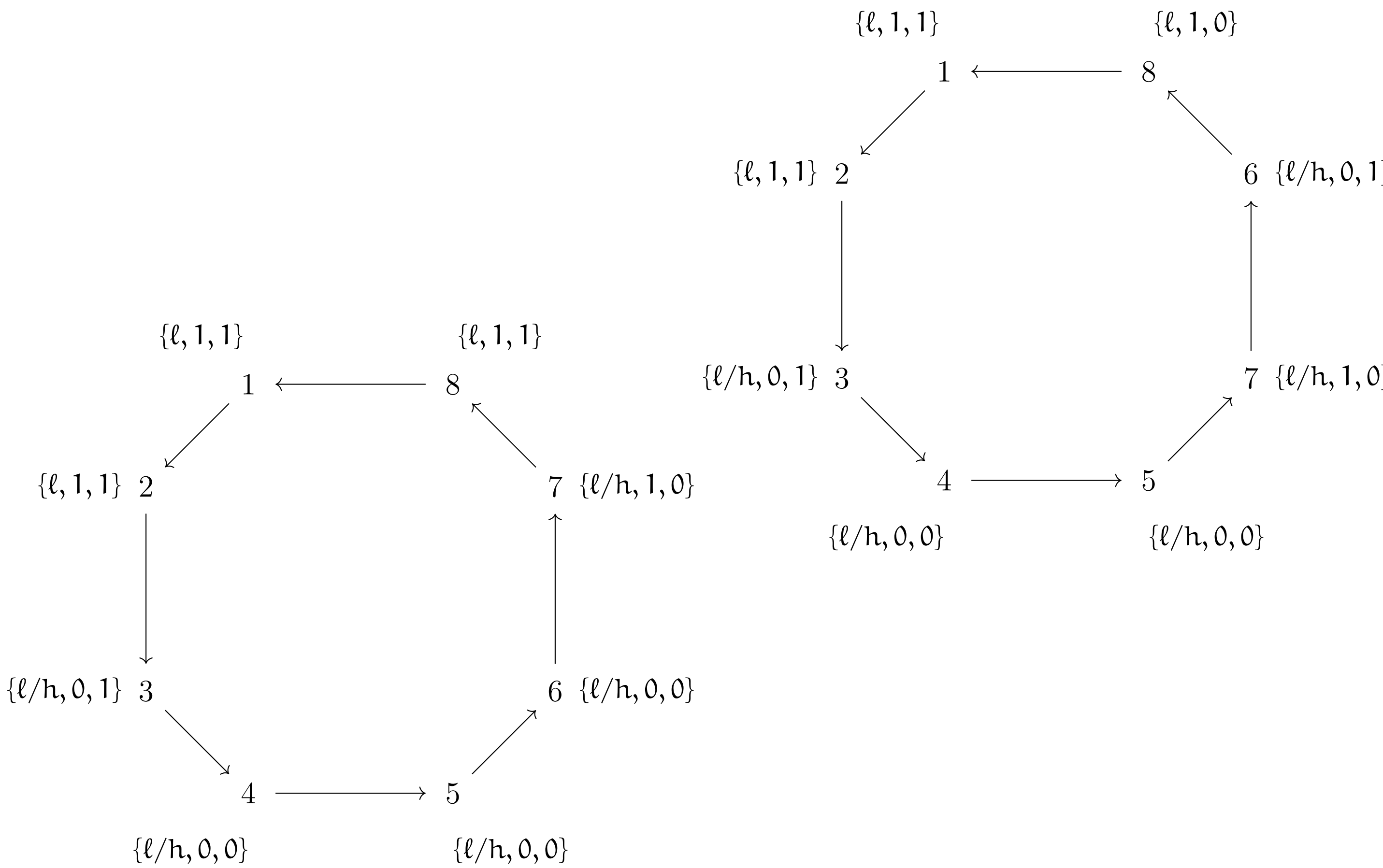

Figure 3: Each agent k knows the profit of his right-hand neighbor (facing away from the perimeter) $k-1$ clockwise, $k = 1, ..., n$ where $0 \equiv n$ for $k = 1$. The entries next to agent k follow the order: $\{\tau_k, y_k, y_{k-1}\}$. The bottom-left panel will involve $m = 3$ (reporting high profit) whereas for the upper-right panel $m = 2$, just by swapping two agents' positions – agent 6 and agent 7. Thus, if the cutoff $\hat{m} = 3$, the bottom-left ordering will trigger audits, whereas the upper-right ordering will lead to no audit. This makes auditing very volatile.

A Appendix: Proofs

Proof of Proposition 1. Write

$$\beta_m = \frac{\alpha}{\alpha + (\theta'/\theta)^m \left(\frac{1-\theta'}{1-\theta}\right)^{n-m} (1-\alpha)}. \tag{10}$$

It is easy to verify that

$$0 < \theta' < \theta < 1, \quad 0 < 1-\theta < 1-\theta' < 1.$$

So, the denominator of the RHS of (10) is decreasing in m, hence β is increasing in m. **Q.E.D.**

Proof of Lemma 1. Let us write out

$$\begin{aligned}\beta_m\mu_1 + (1-\beta_m)\mu_0 &= \beta_m[\mu_1 - \mu_0] + \mu_0 \\ &= \beta_m\left[\frac{A+B}{A+B+C} - \frac{A'+B'}{A'+B'+C'}\right] + \frac{A'+B'}{A'+B'+C'}. \end{aligned} \tag{11}$$

By Proposition 1, β_m is increasing in m. So to show that (11) is increasing in m (i.e., part (ii) of the Lemma), it is enough to establish part (i):

$$\begin{aligned}
&\frac{A+B}{A+B+C} - \frac{A'+B'}{A'+B'+C'} > 0 \\
\text{i.e.,}\quad & (A+B)C' - (A'+B')C > 0 \\
\text{i.e.,}\quad & \frac{\rho_{h,1}q + \rho_{\ell,1}(1-q)[(1-\rho_{h,1})q + (1-\rho_{\ell,1})(1-q)]}{(1-\rho_{h,1})q + (1-\rho_{\ell,1})(1-q)} \\
&> \frac{\rho_{h,0}q + \rho_{\ell,0}(1-q)[(1-\rho_{h,0})q + (1-\rho_{\ell,0})(1-q)]}{(1-\rho_{h,0})q + (1-\rho_{\ell,0})(1-q)} \\
\text{i.e.,}\quad & \frac{\rho_{h,1}q}{(1-\rho_{h,1})q + (1-\rho_{\ell,1})(1-q)} + \rho_{\ell,1}(1-q) \\
&> \frac{\rho_{h,0}q}{(1-\rho_{h,0})q + (1-\rho_{\ell,0})(1-q)} + \rho_{\ell,0}(1-q),
\end{aligned}$$

which is clearly true given that $\rho_{h,1} > \rho_{h,0}$ and $\rho_{\ell,1} > \rho_{\ell,0}$. **Q.E.D.**

■ **Derivation of** $\Pr(y_j = 1, y_{j-1} = 1, \epsilon | \tau_j = \ell, y_k, y_{k-1}, \tau_k)$.

$$\begin{aligned}
& \Pr(y_j = 1, y_{j-1} = 1, \epsilon | \tau_j = \ell, y_k, y_{k-1}, \tau_k) \\
= \; & \frac{\Pr(y_j = 1, y_{j-1} = 1, \epsilon, \tau_j = \ell, y_k, y_{k-1}, \tau_k)}{\Pr(\tau_j = \ell, y_k, y_{k-1}, \tau_k)} \\
= \; & \frac{\Pr(y_j = 1, y_{j-1} = 1, y_k, y_{k-1} | \epsilon, \tau_j = \ell, \tau_k) \Pr(\epsilon) \Pr(\tau_j = \ell) \Pr(\tau_k)}{\Pr(\tau_j = \ell) \Pr(y_k, y_{k-1} | \tau_k) \Pr(\tau_k)} \\
= \; & \frac{\Pr(y_j = 1, y_{j-1} = 1, y_k, y_{k-1} | \epsilon, \tau_j = \ell, \tau_k) \Pr(\epsilon)}{\Pr(y_k, y_{k-1} | \tau_k)}.
\end{aligned}$$

In the numerator,

$$\begin{aligned}
& \Pr(y_j = 1, y_{j-1} = 1, y_k, y_{k-1} | \epsilon, \tau_j = \ell, \tau_k) \\
= \; & \rho_{\ell,\epsilon}[\Pr(y_{j-1} = 1, \tau_{j-1} = h | \epsilon) + \Pr(y_{j-1} = 1, \tau_{j-1} = \ell | \epsilon)] \rho_{\tau_k,\epsilon}[\Pr(y_{k-1}, \tau_{k-1} = h | \epsilon) + \Pr(y_{k-1}, \tau_{k-1} = \ell | \epsilon)] \\
= \; & \rho_{\ell,\epsilon}[\Pr(y_{j-1} = 1 | \tau_{j-1} = h, \epsilon) \Pr(\tau_{j-1} = h) + \Pr(y_{j-1} = 1 | \tau_{j-1} = \ell, \epsilon) \Pr(\tau_{j-1} = \ell)] \\
& \times \rho_{\tau_k,\epsilon}[\Pr(y_{k-1} | \tau_{k-1} = h, \epsilon) \Pr(\tau_{k-1} = h) + \Pr(y_{k-1} | \tau_{k-1} = \ell, \epsilon) \Pr(\tau_{k-1} = \ell)] \\
= \; & \rho_{\ell,\epsilon} \rho_{\tau_k,\epsilon}[\rho_{h,\epsilon} q + \rho_{l,\epsilon}(1 - q)] \cdot [\Pr(y_{k-1} | \tau_{k-1} = h, \epsilon) q + \Pr(y_{k-1} | \tau_{k-1} = \ell, \epsilon)(1 - q)],
\end{aligned}$$

and the denominator can be expressed as

$$\begin{aligned}
& \Pr(y_k, y_{k-1} | \tau_k) \\
= \; & \Pr(y_k, y_{k-1} | \epsilon = 1, \tau_k) \Pr(\epsilon = 1) + \Pr(y_k, y_{k-1} | \epsilon = 0, \tau_k) \Pr(\epsilon = 0) \\
= \; & \Pr(y_k | \epsilon = 1, \tau_k) \Pr(y_{k-1} | \epsilon = 1) \Pr(\epsilon = 1) + \Pr(y_k | \epsilon = 0, \tau_k) \Pr(y_{k-1} | \epsilon = 0) \Pr(\epsilon = 0) \\
= \; & \rho_{\tau_k,1}[\Pr(y_{k-1} | \tau_{k-1} = h, \epsilon = 1) q + \Pr(y_{k-1} | \tau_{k-1} = \ell, \epsilon = 1)(1 - q)] \alpha \\
& + \rho_{\tau_k,0}[\Pr(y_{k-1} | \tau_{k-1} = h, \epsilon = 0) q + \Pr(y_{k-1} | \tau_{k-1} = \ell, \epsilon = 0)(1 - q)](1 - \alpha).
\end{aligned}$$

After we substitute these two expressions, we can get equation (8). ||

■ **Detailed Analysis of Taxpayers' problem**

Case (2): $y_k = 1, y_{k-1} = 0, \tau_k = \ell$.

Next obtain:

$$
\begin{aligned}
&\Pr(y_j = 1, y_{j-1} = 1, \epsilon = 1 | \tau_j = \ell, y_k = 1, y_{k-1} = 0, \tau_k = \ell) \\
= \ & \frac{\rho_{\ell,1}^2[\rho_{\ell,1}(1-q) + \rho_{h,1}q] \cdot [(1-\rho_{\ell,1})(1-q) + (1-\rho_{h,1})q]\alpha}{\rho_{\ell,1}[(1-\rho_{\ell,1})(1-q) + (1-\rho_{h,1})q]\alpha + \rho_{\ell,0}[(1-\rho_{\ell,0})(1-q) + (1-\rho_{h,0})q](1-\alpha)} \\
= \ & \frac{\rho_{\ell,1}^2 T_1 \cdot T_2 \alpha}{\rho_{\ell,1} T_2 \alpha + \rho_{\ell,0} T_4 (1-\alpha)},
\end{aligned}
$$

and

$$
\begin{aligned}
&\Pr(y_j = 1, y_{j-1} = 1, \epsilon = 0 | \tau_j = \ell, y_k = 1, y_{k-1} = 0, \tau_k = \ell) \\
= \ & \frac{\rho_{\ell,0}^2[\rho_{\ell,0}(1-q) + \rho_{h,0}q] \cdot [(1-\rho_{\ell,0})(1-q) + (1-\rho_{h,0})q](1-\alpha)}{\rho_{\ell,1}[(1-\rho_{\ell,1})(1-q) + (1-\rho_{h,1})q]\alpha + \rho_{\ell,0}[(1-\rho_{\ell,0})(1-q) + (1-\rho_{h,0})q](1-\alpha)} \\
= \ & \frac{\rho_{\ell,0}^2 T_3 \cdot T_4 (1-\alpha)}{\rho_{\ell,1} T_2 \alpha + \rho_{\ell,0} T_4 (1-\alpha)}.
\end{aligned}
$$

Let us now define

$$
\eta_{1,0,\ell} \equiv \Pr(\hat{y}_j = 1 \big| y_k = 1, y_{k-1} = 0, \tau_k = \ell) = \frac{\rho_{\ell,1}^2 T_1 \cdot T_2 \alpha + \rho_{\ell,0}^2 T_3 \cdot T_4 (1-\alpha)}{\rho_{\ell,1} T_2 \alpha + \rho_{\ell,0} T_4 (1-\alpha)} \times (1-q).
$$

Now we need to induce $(y_k = 1, y_{k-1} = 0, \tau_k = \ell)$ to submit low profit, which requires the following condition:

$$
\begin{aligned}
&\Pr(m \geq \hat{m} | y_k = 1, y_{k-1} = 0, \tau_k = \ell)(t+f) \leq t \\
\text{i.e.,} \quad &\left[1 - \sum_{i=0}^{\hat{m}-1} \frac{n!}{i!(n-i)!} \eta_{1,0,\ell}^i (1-\eta_{1,0,\ell})^{n-i}\right](t+f) \leq t.
\end{aligned}
$$

Case (3): $y_k = 1, y_{k-1} = 1, \tau_k = h$.

Similarly, derive

$$
\begin{aligned}
& \Pr(y_j = 1, y_{j-1} = 1, \epsilon = 1 | \tau_j = \ell, y_k = 1, y_{k-1} = 1, \tau_k = h) \\
= \; & \frac{\rho_{\ell,1}\rho_{h,1}\left[\rho_{\ell,1}(1-q) + \rho_{h,1}q\right]^2 \alpha}{\rho_{h,1}[\rho_{\ell,1}(1-q) + \rho_{h,1}q]\alpha + \rho_{h,0}[\rho_{\ell,0}(1-q) + \rho_{h,0}q](1-\alpha)} \\
= \; & \frac{\rho_{\ell,1}\rho_{h,1}T_1^2\alpha}{\rho_{h,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)},
\end{aligned}
$$

and

$$
\begin{aligned}
& \Pr(y_j = 1, y_{j-1} = 1, \epsilon = 0 | \tau_j = \ell, y_k = 1, y_{k-1} = 1, \tau_k = h) \\
= \; & \frac{\rho_{\ell,0}\rho_{h,0}\left[\rho_{\ell,0}(1-q) + \rho_{h,0}q\right]^2 (1-\alpha)}{\rho_{h,1}[\rho_{\ell,1}(1-q) + \rho_{h,1}q]\alpha + \rho_{h,0}[\rho_{\ell,0}(1-q) + \rho_{h,0}q](1-\alpha)} \\
= \; & \frac{\rho_{\ell,0}\rho_{h,0}T_3^2(1-\alpha)}{\rho_{h,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)}.
\end{aligned}
$$

Thus, we have

$$
\eta_{1,1,h} \equiv \Pr(\hat{y}_j = 1 \big| y_k = 1, y_{k-1} = 1, \tau_k = h) = \frac{\rho_{\ell,1}\rho_{h,1}T_1^2\alpha + \rho_{\ell,0}\rho_{h,0}T_3^2(1-\alpha)}{\rho_{h,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)} \times (1-q).
$$

Underreporting by agent k with $(y_k = 1, y_{k-1} = 1, \tau_k = h)$ requires:

$$
\begin{aligned}
& \Pr(m \geq \hat{m} | y_k = 1, y_{k-1} = 1, \tau_k = h)(t+f) \leq t \\
\text{i.e.,} \quad & \left[1 - \sum_{i=0}^{\hat{m}-1} \frac{n!}{i!(n-i)!}\eta_{1,1,h}^i(1-\eta_{1,1,h})^{n-i}\right](t+f) \leq t.
\end{aligned}
$$

Case (4): $y_k = 1, y_{k-1} = 0, \tau_k = h$.

Finally, let us derive

$$
\begin{aligned}
& \Pr(y_j = 1, y_{j-1} = 1, \epsilon = 1 | \tau_j = \ell, y_k = 1, y_{k-1} = 0, \tau_k = h) \\
= \; & \frac{\rho_{\ell,1}\rho_{h,1}\left[\rho_{\ell,1}(1-q) + \rho_{h,1}q\right]\left[(1-\rho_{\ell,1})(1-q) + (1-\rho_{h,1})q\right]\alpha}{\rho_{h,1}[(1-\rho_{\ell,1})(1-q) + (1-\rho_{h,1})q]\alpha + \rho_{h,0}[(1-\rho_{\ell,0})(1-q) + (1-\rho_{h,0})q](1-\alpha)}
\end{aligned}
$$

$$= \frac{\rho_{\ell,1}\rho_{h,1}T_1 \cdot T_2\alpha}{\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)},$$

and

$$\begin{aligned}
&\Pr(y_j = 1, y_{j-1} = 1, \epsilon = 0 | \tau_j = \ell, y_k = 1, y_{k-1} = 0, \tau_k = h) \\
= &\frac{\rho_{\ell,0}\rho_{h,0}\left[\rho_{\ell,0}(1-q) + \rho_{h,0}q\right]\left[(1-\rho_{\ell,0})(1-q) + (1-\rho_{h,0})q\right](1-\alpha)}{\rho_{h,1}[(1-\rho_{\ell,1})(1-q) + (1-\rho_{h,1})q]\alpha + \rho_{h,0}[(1-\rho_{\ell,0})(1-q) + (1-\rho_{h,0})q](1-\alpha)} \\
= &\frac{\rho_{\ell,0}\rho_{h,0}T_3 \cdot T_4(1-\alpha)}{\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)}.
\end{aligned}$$

So,

$$\eta_{1,0,h} \equiv \Pr(\hat{y}_j = 1 \big| y_k = 1, y_{k-1} = 0, \tau_k = h) = \frac{\rho_{\ell,1}\rho_{h,1}T_1 \cdot T_2\alpha + \rho_{\ell,0}\rho_{h,0}T_3 \cdot T_4(1-\alpha)}{\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)} \times (1-q).$$

Again, underreporting by agent k with $(y_k = 1, y_{k-1} = 0, \tau_k = h)$ requires:

$$\begin{aligned}
&\Pr(m \geq \hat{m} | y_k = 1, y_{k-1} = 0, \tau_k = h)(t+f) \leq t \\
\text{i.e.,} \quad &\left[1 - \sum_{i=0}^{\hat{m}-1} \frac{n!}{i!(n-i)!}\eta_{1,0,h}^i(1-\eta_{1,0,h})^{n-i}\right](t+f) \leq t.
\end{aligned}$$

This part ends. ||

Proof of Lemma 2. **1.** First, let us compare $\eta_{1,1,\ell}$ and $\eta_{1,0,\ell}$.

$$\begin{aligned}
&\eta_{1,1,\ell} - \eta_{1,0,\ell} \\
= &\frac{(\rho_{\ell,1}T_1)^2\alpha + (\rho_{\ell,0}T_3)^2(1-\alpha)}{\rho_{\ell,1}T_1\alpha + \rho_{\ell,0}T_3(1-\alpha)} \times (1-q) - \frac{\rho_{\ell,1}^2T_1 \cdot T_2\alpha + \rho_{\ell,0}^2T_3 \cdot T_4(1-\alpha)}{\rho_{\ell,1}T_2\alpha + \rho_{\ell,0}T_4(1-\alpha)} \times (1-q).
\end{aligned}$$

After factoring out $(1-q)$ and simplifying, the denominator

$$[\rho_{\ell,1}T_1\alpha + \rho_{\ell,0}T_3(1-\alpha)]\,[\rho_{\ell,1}T_2\alpha + \rho_{\ell,0}T_4(1-\alpha)]$$

is always positive. The numerator (excluding the factor $(1-q)$) is

$$\begin{aligned}
&\left[(\rho_{\ell,1}T_1)^2\alpha + (\rho_{\ell,0}T_3)^2(1-\alpha)\right]\left[\rho_{\ell,1}T_2\alpha + \rho_{\ell,0}T_4(1-\alpha)\right] \\
&- \left[\rho_{\ell,1}^2 T_1\cdot T_2\alpha + \rho_{\ell,0}^2 T_3\cdot T_4(1-\alpha)\right]\left[\rho_{\ell,1}T_1\alpha + \rho_{\ell,0}T_3(1-\alpha)\right] \\
=\ & \alpha(1-\alpha)\rho_{\ell,0}\rho_{\ell,1}(\rho_{\ell,0}T_2T_3^2 + \rho_{\ell,1}T_1^2T_4 - \rho_{\ell,0}T_1T_3T_4 - \rho_{\ell,1}T_1T_2T_3) \\
=\ & \alpha(1-\alpha)\rho_{\ell,0}\rho_{\ell,1}(\rho_{\ell,1}T_1 - \rho_{\ell,0}T_3)(T_1T_4 - T_2T_3) > 0,
\end{aligned}$$

since $\rho_{\ell,1} > \rho_{\ell,0}$, $T_1 > T_3$ and $T_4 > T_2$. Thus, $\eta_{1,1,\ell} > \eta_{1,0,\ell}$.

Next, let us compare $\eta_{1,0,\ell}$ and $\eta_{1,0,h}$.

$$\begin{aligned}
&\eta_{1,0,\ell} - \eta_{1,0,h} \\
=\ & \frac{\rho_{\ell,1}^2 T_1\cdot T_2\alpha + \rho_{\ell,0}^2 T_3\cdot T_4(1-\alpha)}{\rho_{\ell,1}T_2\alpha + \rho_{\ell,0}T_4(1-\alpha)}\times(1-q) - \frac{\rho_{\ell,1}\rho_{h,1}T_1\cdot T_2\alpha + \rho_{\ell,0}\rho_{h,0}T_3\cdot T_4(1-\alpha)}{\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)}\times(1-q).
\end{aligned}$$

The denominator is

$$\left[\rho_{\ell,1}T_2\alpha + \rho_{\ell,0}T_4(1-\alpha)\right]\left[\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)\right],$$

which is always positive. The numerator (excluding the factor $(1-q)$) is

$$\begin{aligned}
&\left[\rho_{\ell,1}^2 T_1\cdot T_2\alpha + \rho_{\ell,0}^2 T_3\cdot T_4(1-\alpha)\right]\left[\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)\right] \\
&- \left[\rho_{\ell,1}\rho_{h,1}T_1\cdot T_2\alpha + \rho_{\ell,0}\rho_{h,0}T_3\cdot T_4(1-\alpha)\right]\left[\rho_{\ell,1}T_2\alpha + \rho_{\ell,0}T_4(1-\alpha)\right] \\
=\ & \alpha(1-\alpha)T_2T_4(\rho_{\ell,0}^2\rho_{h,1}T_3 + \rho_{\ell,1}^2\rho_{h,0}T_1 - \rho_{\ell,0}\rho_{\ell,1}\rho_{h,1}T_1 - \rho_{\ell,0}\rho_{\ell,1}\rho_{h,0}T_3) \\
=\ & \alpha(1-\alpha)T_2T_4(\rho_{\ell,1}T_1 - \rho_{\ell,0}T_3)(\rho_{\ell,1}\rho_{h,0} - \rho_{\ell,0}\rho_{h,1}) > 0,
\end{aligned}$$

since $\rho_{\ell,1} > \rho_{\ell,0}$, $T_1 > T_3$ and $\rho_{\ell,1}\rho_{h,0} > \rho_{\ell,0}\rho_{h,1}$ by Assumption 1. Thus, $\eta_{1,0,\ell} > \eta_{1,0,h}$.

2. Now let us compare $\eta_{1,1,h}$ and $\eta_{1,0,h}$.

$$\begin{aligned}&\eta_{1,1,h} - \eta_{1,0,h}\\ = &\frac{\rho_{\ell,1}\rho_{h,1}T_1^2\alpha + \rho_{\ell,0}\rho_{h,0}T_3^2(1-\alpha)}{\rho_{h,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)} \times (1-q) - \frac{\rho_{\ell,1}\rho_{h,1}T_1 \cdot T_2\alpha + \rho_{\ell,0}\rho_{h,0}T_3 \cdot T_4(1-\alpha)}{\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)} \times (1-q).\end{aligned}$$

The denominator

$$[\rho_{\ell,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)]\,[\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)]$$

is always positive. The numerator (excluding the factor $(1-q)$) is

$$\begin{aligned}&\left[\rho_{\ell,1}\rho_{h,1}T_1^2\alpha + \rho_{\ell,0}\rho_{h,0}T_3^2(1-\alpha)\right]\,[\rho_{h,1}T_2\alpha + \rho_{h,0}T_4(1-\alpha)]\\ &-[\rho_{\ell,1}\rho_{h,1}T_1 \cdot T_2\alpha + \rho_{\ell,0}\rho_{h,0}T_3 \cdot T_4(1-\alpha)]\,[\rho_{h,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)]\\ = \;&\alpha(1-\alpha)\rho_{h,1}\rho_{h,0}(\rho_{\ell,0}T_2T_3^2 + \rho_{\ell,1}T_1^2T_4 - \rho_{\ell,0}T_1T_3T_4 - \rho_{\ell,1}T_1T_2T_3)\\ = \;&\alpha(1-\alpha)\rho_{h,1}\rho_{h,0}(\rho_{\ell,1}T_1 - \rho_{\ell,0}T_3)(T_1T_4 - T_2T_3) > 0,\end{aligned}$$

since $\rho_{\ell,1} > \rho_{\ell,0}$, $T_1 > T_3$ and $T_4 > T_2$. Thus, $\eta_{1,1,h} > \eta_{1,0,h}$.

Finally, let us compare $\eta_{1,1,\ell}$ and $\eta_{1,1,h}$.

$$\begin{aligned}&\eta_{1,1,\ell} - \eta_{1,1,h}\\ = &\frac{\rho_{\ell,1}^2T_1^2\alpha + \rho_{\ell,0}^2T_3^2(1-\alpha)}{\rho_{\ell,1}T_1\alpha + \rho_{\ell,0}T_3(1-\alpha)} \times (1-q) - \frac{\rho_{\ell,1}\rho_{h,1}T_1^2\alpha + \rho_{\ell,0}\rho_{h,0}T_3^2(1-\alpha)}{\rho_{h,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)} \times (1-q).\end{aligned}$$

The denominator

$$[\rho_{\ell,1}T_1\alpha + \rho_{\ell,0}T_3(1-\alpha)]\,[\rho_{h,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)]$$

is always positive. The numerator (excluding the factor $(1-q)$) is

$$\begin{aligned}
&\left[\rho_{\ell,1}^2 T_1^2\alpha + \rho_{\ell,0}^2 T_3^2(1-\alpha)\right]\left[\rho_{h,1}T_1\alpha + \rho_{h,0}T_3(1-\alpha)\right] \\
&- \left[\rho_{\ell,1}\rho_{h,1}T_1^2\alpha + \rho_{\ell,0}\rho_{h,0}T_3^2(1-\alpha)\right]\left[\rho_{\ell,1}T_1\alpha + \rho_{\ell,0}T_3(1-\alpha)\right] \\
=\ & \alpha(1-\alpha)T_1T_3(\rho_{\ell,0}^2\rho_{h,1}T_3 + \rho_{\ell,1}^2\rho_{h,0}T_1 - \rho_{\ell,0}\rho_{\ell,1}\rho_{h,1}T_1 - \rho_{\ell,0}\rho_{\ell,1}\rho_{h,0}T_3) \\
=\ & \alpha(1-\alpha)T_1T_3(\rho_{\ell,1}T_1 - \rho_{\ell,0}T_3)(\rho_{\ell,1}\rho_{h,0} - \rho_{\ell,0}\rho_{h,1}) > 0,
\end{aligned}$$

since $\rho_{\ell,1} > \rho_{\ell,0}$, $T_1 > T_3$ and $\rho_{\ell,1}\rho_{h,0} > \rho_{\ell,0}\rho_{h,1}$ by Assumption 1. Thus, $\eta_{1,1,\ell} > \eta_{1,1,h}$. **Q.E.D.**

Proof of Lemma 3. It is equivalent to showing the following:

$$\sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,1,\ell}^i(1-\eta_{1,1,\ell})^{n-i} < \sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,0,\ell}^i(1-\eta_{1,0,\ell})^{n-i}, \tag{12}$$

$$\sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,1,\ell}^i(1-\eta_{1,1,\ell})^{n-i} < \sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,0,h}^i(1-\eta_{1,0,h})^{n-i}, \tag{13}$$

and
$$\sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,1,\ell}^i(1-\eta_{1,1,\ell})^{n-i} < \sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}\eta_{1,1,h}^i(1-\eta_{1,1,h})^{n-i}. \tag{14}$$

Define

$$D \equiv \sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}y^i(1-y)^{n-i} - \sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}x^i(1-x)^{n-i}, \quad 0 < x, y < 1.$$

Note that $D = 0$ for $x = y$.

Next observe that

$$\begin{aligned}
&\frac{\partial \sum_{i=0}^{\hat{m}-1}\frac{n!}{i!(n-i)!}x^i(1-x)^{n-i}}{\partial x} \\
&= \sum_{i=0}^{\hat{m}-1} i \times \frac{n!}{i!(n-i)!}x^{i-1}(1-x)^{n-i} - (n-i)\times\frac{n!}{i!(n-i)!}x^i(1-x)^{n-i-1} \\
&= \sum_{i=0}^{\hat{m}-1} i \times \frac{n!}{i!(n-i)!}x^{i-1}(1-x)^{n-i-1}\left[(1-x) - \frac{1}{i}(n-i)x\right]
\end{aligned}$$

$$= \sum_{i=0}^{\hat{m}-1} \frac{n!}{i!(n-i)!} x^{i-1}(1-x)^{n-i-1}(i - xn) < 0,$$

$$\text{if} \quad \hat{m} - 1 < xn.$$

Thus, as long as $\hat{m} < xn + 1$, when $y > x$, we can get $D < 0$.

Since $\eta_{1,1,\ell} > \eta_{1,0,\ell}$ (by part 1 of Lemma 2), the sufficient condition

$$\hat{m} < \eta_{1,0,\ell} n + 1 \tag{15}$$

will guarantee that condition (12) is satisfied. Similarly, since $\eta_{1,1,\ell} > \eta_{1,0,h}$ (applying transitivity on part 1 or 2 of Lemma 2) and $\eta_{1,1,\ell} > \eta_{1,1,h}$ (by part 2 of Lemma 2), the sufficient conditions

$$\hat{m} < \eta_{1,0,h} n + 1 \tag{16}$$

$$\text{and} \quad \hat{m} < \eta_{1,1,h} n + 1 \tag{17}$$

will guarantee that conditions (13) and (14) are satisfied.

Since $\eta_{1,0,h} < \eta_{1,0,\ell}$ and $\eta_{1,0,h} < \eta_{1,1,h}$ (by Lemma 2), conditions (15), (16), and (17) can be reduced to just one sufficient condition (16). **Q.E.D.**

Proof of Proposition 2. The equilibrium requires finding a cutoff $\hat{m} \in \{1, ..., n-1\}$ such that (i) the auditor will find it optimal to audit all low tax returns if the number of low returns $m \geq \hat{m}$ and otherwise not audit, and (ii) only type $(1, 1, \ell)$ taxpayers submit high reports.

We first consider the auditor's strategy. Suppose $c_0 < c \leq c_1$. Since $\left[\beta_1\mu_1 + (1 - \beta_1)\mu_0\right](t+f) = c_1 \geq c$, it is optimal for the tax authority to audit when $m = 1$. By Lemma 1, the tax authority should audit for all $m \geq 1$. When $m = 0$, since $\left[\beta_0\mu_1 + (1-\beta_0)\mu_0\right](t+f) = c_0 < c$, the tax authority will not audit.

Suppose $c_1 < c \leq c_2$. Since $\left[\beta_2\mu_1 + (1 - \beta_2)\mu_0\right](t + f) = c_2 \geq c$, it is optimal for the tax authority to audit when $m = 2$. By Lemma 1, the tax authority should audit for all $m \geq 2$.

When $m = 1$, since $\left[\beta_1\mu_1 + (1 - \beta_1)\mu_0\right](t + f) = c_1 < c$, the tax authority will not audit. By Lemma 1, the tax authority should not audit for $m < 1$ either.

We can use similar arguments to show that the tax authority's optimal auditing strategy for different c values are as described.

Now we look at the taxpayers' ICs. For any $\hat{m} \leq m^*$, by Lemma 3, we know $\frac{f}{t+f}\big|_{\min}(\hat{m}) < \frac{f}{t+f}\big|_{\max}(\hat{m})$. Since t and f satisfying $\frac{f}{t+f}\big|_{\min}(\hat{m}) \leq \frac{f}{t+f} \leq \frac{f}{t+f}\big|_{\max}(\hat{m})$, the taxpayer's ICs are satisfied, so that they will choose evasion strategies according to (3). **Q.E.D.**

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